Using a quadratic IP:

$$f'(x_0) \approx p'[f \mid x_{-1}, x_0, x_1] = \frac{f(x_1) - f(x_{-1})}{x_1 - x_{-1}}$$

$$= \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$f''(x_0) \approx p''[f \mid x_{-1}, x_0, x_1] = \frac{f(x_1) - 2f(x_0) + f(x_{-1})}{h^2}$$

$$= \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

(so-called centered FD)

Ex.: (5) Approx. first derivative of $f(x) = \sin(x)$ at $x = 1.2$ (exact $f'(x) = \cos(x)$ of course :-)

$\rightsquigarrow$ Slides

We observe

(i) The error $e = \left| p'[f|\ldots] - f'(x) \right|$ behaves as $\begin{Bmatrix} \mathcal{O}(h) \\ \mathcal{O}(h^2) \end{Bmatrix}$ for $\begin{Bmatrix} \text{forward/backward} \\ \text{centered} \end{Bmatrix}$ FD

(ii) The error grows if h is too small

round-off errors

Why? Due to the finite precision of floating point numbers