

The error estimator could be derived from the interpolation error... But there is another way with the help of Taylor expansions/series:

$$f(x+h) = f(x) + f'(x) \cdot h + \frac{f''(x)}{2} h^2 + \dots + \frac{f^{(k)}(x)}{k!} h^k + \frac{f^{(k+1)}(\xi)}{(k+1)!} h^{k+1}$$

↑ for some  $\xi \in [x, x+h]$   
 remainder term  $\rightsquigarrow$  (sometimes error term)

Ex.: (6) forward FD approx of  $f'$ :

$$\frac{f(x_0+h) - f(x_0)}{h} = \frac{\cancel{f(x_0)} + f'(x_0) \cdot h + \frac{f''(x_0)}{2} h^2 + \dots - \cancel{f(x_0)}}{h}$$

$$= f'(x_0) + \frac{h}{2} f''(x_0) + \dots \quad ? \text{negligible}$$

$$= f'(x_0) + \mathcal{O}(h) \quad \checkmark$$

$h > h^2 > h^3$   
 for  $h \ll 1$   
 small

So forward FD has order of accuracy  $\nu = 1$ . They are first order accurate