In the following we will only see numerical methods for first order IPV.

Reduction to first order system

Given an n-th order ODE

$$y^{(n)}(t) = f(t, y(t), \dot{y}(t), \ddot{y}(t), \dots, y^{(n-1)}(t))$$

We define

$$\left.\begin{aligned}
z_0(t) &= y(t) \\
z_1(t) &= \dot{y}(t) = \dot{z}_0(t) \\
z_2(t) &= \ddot{y}(t) = \dot{z}_1(t) \\
&\vdots \\
z_{n-1}(t) &= y^{(n-1)}(t) = \dot{z}_{n-2}(t)
\end{aligned}\right\} \text{ $n-1$ first order ODEs}$$

Now

$$\begin{aligned}
z_n(t) = y^{(n)}(t) &= \underline{\dot{z}_{n-1}(t)} \\
&= f(t, y(t), \dot{y}(t), \ddot{y}(t), \dots, y^{(n-1)}(t)) \\
&= \underline{f(t, z_0(t), z_1(t), z_2(t), \dots, z_{n-1}(t))}
\end{aligned}$$

$\rightsquigarrow$ n first order ODEs!