III.3 Error estimation and convergence

To analyze the accuracy of so-called one-step methods we consider the general expression

$$y_{j+1} = y_j + h \cdot \phi(t_j, y_j, y_{j+1}, h)$$

and call ϕ the increment function.

Rem.: (i) $\phi(t_j, y_j, y_{j+1}, h) = f(t_j, y_j)$ EE

(ii) $\phi(t_j, y_j, y_{j+1}, h) = f(t_{j+1}, y_{j+1})$ IE

Def.: the local truncation error (LTE) at t_j is defined by

$$e_j = y(t_j) - \left(y(t_{j-1}) + h \cdot \phi(t_{j-1}, y(t_{j-1}), y(t_j), h) \right)$$

exact sol. at t_j t_{j-1}

The LTE is the error after one step of a method executed with the exact solution.

Def.: the global truncation error (GTE) at t_j is defined by

$$\varepsilon_j = y(t_j) - y_j$$

exact approx. solution at t_j