

Ex.: (8) LTE of IE $\phi(t_j, y_j, y_{j+1}, h) = f(t_{j+1}, y_{j+1})$

$$e_j \stackrel{\text{def.}}{=} y(t_j) - y(t_{j-1}) - h \cdot f(t_j, y(t_j))$$

$$= \dots$$

$$= \mathcal{O}(h^2)$$

$y(t_{j-1}) = \dots$ Taylor...

Rem.: The LTE depends on the smoothness of the solution (e.g. y above!) and therefore on the smoothness of the right-hand side function $f(t, y(t))$:

$$\ddot{y}(t) = \frac{d}{dt}(\dot{y}(t))$$

$$\stackrel{\text{ODE}}{=} \frac{d}{dt}(f(t, y(t)))$$

$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} \frac{dy}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} f$$

$$\text{ODE } \dot{y} = f(t, y)$$

So we have seen that the LTE error is easy to compute with Taylor series...

How is the LTE related to the practically more meaningful CTE?