Ex.: (9) EE & IE GTE

⇝ Slides (Euler Methods)

We observe that although the LTE behaves as $\mathcal{O}(h^2)$, the GTE is $\mathcal{O}(h)$

Actually, one can show (under requirements that are anyway necessary for the existence and uniqueness of solutions to an IVP) that the LTEs accumulate in each step:

$$|E_N| \le \underbrace{N}_{\sim \frac{1}{h}} \cdot \max_{1 \le j \le N} \underbrace{|e_j|}_{\mathcal{O}(h^2)} = \mathcal{O}(h)$$

And we have convergence: $y_N \xrightarrow{h \to 0} y(T)$

This motivates the following definition

Def.: We say that a method has order of accuracy p if the LTE is

$$|e_j| = \mathcal{O}(h^{p+1})$$

$p+1$!

or equivalently

$$|E_j| = \mathcal{O}(h^p)$$

Rem.: EE and IE have order of accuracy $p=1$.