

III.5 Absolute stability

Let's consider the very simple IVP

$$\begin{aligned} \dot{y}(t) &= \lambda y(t) & (\lambda \in \mathbb{R} \text{ or } \mathbb{C}) \\ y(0) &= y_0 \end{aligned}$$

which is known as the Dahlquist test equation (DTE) (in the present context).

The analytical solution is simply

$$y(t) = y_0 e^{\lambda t}$$

Let's apply EE to this IVP

$$\begin{aligned} y_{j+1} &= y_j + h \cdot f(t_j, y_j) \\ &= y_j + h\lambda y_j \\ &= (\lambda + h\lambda) y_j \\ &= (\lambda + h\lambda)^2 y_{j-1} & \left. \begin{array}{l} y_j = (\lambda + h\lambda) y_{j-1} \\ y_{j-1} = (\lambda + h\lambda) y_{j-2} \end{array} \right\} \\ &\vdots \\ &= (\lambda + h\lambda)^{j+1} y_0 \end{aligned}$$