

IV. Partial Differential Equations

Practical problems often depend on more than one variable, e.g. time (t) and space (x, y, z).

This leads to so-called Partial Differential Equations (PDEs).

Classical examples:

$$(i) \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{Laplace eq.}$$

$$(ii) \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y) \quad \text{Poisson eq.}$$

$$(iii) \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{Heat eq. (Diffusion)}$$

$$(iv) \quad \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{Wave eq.}$$

In order to form well-posed problems, PDEs have to be supplemented by appropriate boundary conditions and sometimes also with initial conditions.

The above (linear) PDEs can be classified as
elliptic, parabolic or hyperbolic
/ / /
steady states diffusion processes wave phenomena

Unlike ODEs, PDEs can not be analyzed "uniformly" and there is no general PDE solver.

Numerical methods for PDE is a very vast subject and a vast area of research.

Given the limited amount of time, we look only at few examples...

Ex.: (1) Tubular reactor \rightsquigarrow slides

(2) Steady state tubular reactor \rightsquigarrow slides

(3) Time dependent tubular reactor

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on 13.11.24

Once more: this is a vast subject and we only glanced at the subject

Other methods: finite volume

finite element

Discontinuous Galerkin (DG)