IV. Partial Differential Equations

Practical problems often depend on more than one variable, e.g. time (t) and space (x, y, z).
This leads to so-called Partial Differential Equations (PDEs).

Classical examples:

(i) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Laplace eq.

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ Poisson eq.

(iii) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ Heat eq. (Diffusion)

(iv) $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ Wave eq.

In order to form well-posed problems, PDEs have to be supplemented by appropriate boundary conditions and sometimes also with initial conditions.

The above (linear) PDEs can be classified as elliptic, parabolic or hyperbolic

steady state — diffusion processes — wave phenomena