Least-squares solution:

$$\min_{\vec{x} \in D} \phi(\vec{x})$$

where $\phi(\vec{x}) = \frac{1}{2} \| \vec{f}(\vec{x}) - \vec{b} \|_2^2$

Let's rewrite $\phi(\vec{x})$:

$$\phi(\vec{x}) = \frac{1}{2} \| \vec{f}(\vec{x}) - \vec{b} \|_2^2$$

$$= \frac{1}{2} \sum_{i=1}^{m} \left(f_i(\vec{x}) - b_i \right)^2$$

(not sufficient ... see below)

A <u>necessary condition</u> for a minimum is that the gradient of ϕ vanishes:

$$\vec{\nabla} \phi = \begin{pmatrix} \frac{\partial \phi}{\partial x_1} \\ \vdots \\ \frac{\partial \phi}{\partial x_n} \end{pmatrix} \overset{!}{=} 0$$

Then $$\frac{\partial \phi}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{1}{2} \sum_{i=1}^{m} \left(f_i(x_1, \ldots, x_j, \ldots, x_n) - b_i \right)^2 \right)$$

$$= \sum_{i=1}^{m} \left(f_i(\vec{x}) - b_i \right) \cdot \frac{\partial f_i}{\partial x_j} \overset{!}{=} 0 \,, \quad j = 1, \ldots, n$$

These are n (nonlinear) equations for the n unknown parameters