V.2.2 Gauss-Newton method

<u>Idea:</u> linearize the residual equations $\vec{F}(\vec{x})-\vec{b}$ and solve a sequence of linear least-squares problems

k-th iteration

So linearize at some given $\vec{x}^{(k)}$:

$$\vec{F}(\vec{x})-\vec{b} \approx \vec{F}(\vec{x}^{(k)})-\vec{b} + J(\vec{x}^{(k)})(\vec{x}-\vec{x}^{(k)})$$

$$\rightsquigarrow \quad A^{(k)}\vec{x} - \vec{\beta}^{(k)}$$

Simply rearranging the terms... and a linear least-squares problem appears!

So instead of solving the nonlinear least-squares problem, one solves a sequence of linear least-squares problems:

$$\vec{x}^{(0)} \longrightarrow \min_{\vec{x}\in D} \frac{1}{2}\|A^{(0)}\vec{x}-\vec{\beta}^{(0)}\|_2^2 \longrightarrow \vec{x}^{(1)}$$

initial guess

$$\min_{\vec{x}\in D} \frac{1}{2}\|A^{(1)}\vec{x}-\vec{\beta}^{(1)}\|_2^2 \longrightarrow \vec{x}^{(2)}$$

$$\vdots$$

$$\min_{\vec{x}\in D} \frac{1}{2}\|A^{(k)}\vec{x}-\vec{\beta}^{(k)}\|_2^2 \longrightarrow \vec{x}^{(k+1)}$$

Stop when $\|\vec{x}^{(k+1)}-\vec{x}^{(k)}\| < \text{tolerance}$