CORRECTIONS TO "AN INTRODUCTION TO PROBABILISTIC NUMBER THEORY"

EMMANUEL KOWALSKI

Corrections

Here are the currently known corrections; those which are mathematically significant are marked with a star. The page numbers refer to the published version.

(1) p. xiv, (10): $\bar{x}$ instead of $\bar{q}$.

(2) p. 160, l. -3: the formula should be

$$\widehat{\varphi}(s) = \frac{(-1)^k}{s(s+1)\cdots(s+k-1)} \int_0^{+\infty} \varphi^{(k)}(x) x^{s+k-1} dx.$$

(3) p. 165, Prop. A.4.3: write "Let $\sigma_0 > 0$ be given" (instead of "Let $\sigma > 0$ be given".

(4) ($\star$) p. 165, Prop. A.4.3: the assumption on f should include the fact that $f(s)$ has polynomials growth in vertical strips $\sigma \leqslant \mathrm{Re}(s) \leqslant \mathrm{A}$ for any $\sigma > \sigma_0$.

(5) p. 166, l. 5 and 6: the integral should be over the line with real part α.

(6) p. 166, l. -13: the outcome of Cauchy's formula should be

$$\frac{1}{2i\pi} \int_{\mathcal{R}_{\mathrm{T}}} f(s+w) \mathrm{N}^w \hat{\varphi}(w) dw = f(s) + c\mathrm{N}^{1-s} \hat{\varphi}(1-s).$$

To conclude the proof, one observes that the polynomial growth assumption implies that the horizontal contributions tend to 0 as $\mathrm{T} \to +\infty$, and the contributions on the vertical segment with real part α converges to

$$\frac{1}{2i\pi} \int_{(\alpha)} f(s+w) \mathrm{N}^w \hat{\varphi}(w) dw = f_{\mathrm{N}}(s).$$

(7) p. 172, Section § B.2: as pointed out by O. Gabber, it is not true that any Borel measure on a *compact* topological space has a well-defined support, in the sense (2) in the book, although this is true if the measure is regular (which is the case in all the examples considered in the book). Gabber mentions the following counterexample (attributed to Dieudonné): let $\mathrm{M} = [0, \omega_1]$ be the set of ordinals between 0 and the first uncountable ordinal, with the order topology; for a Borel set E of $[0, \omega_1[= \omega_1$, either E or its complement contains an unbounded closed set; in the first case, let $\mu(\mathrm{E}) = 1$ and in the second case, let $\mu(\mathrm{E}) = 0$. This can be extended to a measure on X; then its support in the sense (2) is $\{\omega_1\}$, but the measure of $\mathrm{X} - \{\omega_1\}$ is equal to 1.

Date: February 27, 2023.

(8) p. 249, reference [80]: this paper has appeared in Advances in Math., 385 (2021).

(9) p. 250, reference [86]: the first author is J. Marklof, and not J. Markov.

Additions

We will list here some additions and other comments that are not corrections properly speaking.

For the moment, there are none.