Monte Verità Winter School
Titles and abstracts

- **E. Kowalski**: *Trace functions: definitions, examples and basic formalism*
  We will discuss the precise defintions and basic formalisms of trace functions. The key result that we will explain is Deligne's general form of the Riemann Hypothesis over finite fields, stated as a quasi- orthogonality property of trace functions. We will present in detail the most important examples, and explain in particular how some famous exponential sums estimates in analytic number theory literature can be derived as straightforward consequences of this formalism.
- **Ph. Michel**: *Some applications of trace functions in analytic number theory*
  In these lecture we will discuss through various examples how general trace functions (considered as q-periodic functions on the integers) correlate with other, naturally defined, arithmetic functions and will describe various methods to evaluate these correlations and this can lead to complete sums involving sometimes quite complicated new trace functions. We will then explain how this related to the general theory as discussed in the lectures of E. Kowalski and W. Sawin and will apply these to various classical problems in analytic number theory, including moments of L-functions.
- **W. Sawin**: *Trace functions beyond the basic formalism*
  Trace functions arise from representations of the Galois group of the field $\mathbf{F}_q(T)$. We will explain how features of these representations give rise to concrete properties of the trace functions, culminating in Deligne's powerful equidistribution theorem, and explain the analogous situation with solutions of ordinary differential equations over the complex numbers to provide some additional intuition. We will discuss some of the most important examples of trace functions and how this relationship plays out in those cases. We will begin to describe some of the methods that are used to calculate with these representations in practice.
- **L. Pierce**: *Short character sums*
  Many problems in analytic number theory hinge upon obtaining suitable upper bounds for character sums, involving either a multiplicative character, an additive character, or both. The method for obtaining such bounds depends strongly on whether the sum is complete (that is, it sums over a complete set of residues), or sums over a shorter interval. As will be treated in other lectures, complete character sums relate closely to trace functions, and in many cases are known to admit a very good upper bound. Less is understood about short character sums; so far one's main hope is to relate them back to complete character sums. If the sums are not too short, Fourier expansion is a useful technique for this. For shorter sums, an ingenious argument of Burgess provides a means to relate back to complete character sums. In these lectures we will recall Burgess's classical argument for multiplicative character sums, and then explore new work on short character sums in more general settings, involving both multiplicative and additive characters, arbitrary dimensions, and nonlinear arguments.
- **T. Browning**: *Diophantine applications of exponential sums*
  The Manin conjecture is concerned with the quantitative density of rational points on algebraic varieties with ample anticanonical divisor. On attaching a suitable height function, the conjecture relates geometric invariants of the variety to the asymptotic behaviour of the function that counts rational points of bounded height on the variety. Recent examples have come to light suggesting that a refinement of the conjecture is required to cope with rational points lying on 'thin sets' in the variety, as introduced to the subject by Serre.

One such refinement, due to Peyre, involves restricting the count to those rational points attaining an adequate measure of 'freeness', which is a notion that comes from birational geometry but can be explained in down to earth terms using the geometry of numbers. One of the most versatile tools for studying rational points is the Hardy-Littlewood circle method. This a Fourier analytic technique that naturally brings properties of exponential sums into focus. In these lectures I will discuss the Manin conjecture through the lens of the circle method, with am emphasis on quadratic forms and Peyre's idea of freeness.

Schedule

- **Monday, January 14.**
  - 8:45 to 9:05: Welcome by CSF and presentation of Monte Verità
  - 9:10 to 10:10: E. Kowalski (1/4)
  - 10:10 to 10:30: Coffee break
  - 10:30 to 11:30: L. Pierce (1/3)
  - 11:30 to 14:00: Lunch
  - 14:00 to 15:00: T. Browning (1/3)
  - 15:30 to 16:30: L. Pierce (2/3)
- **Tuesday, January 15.**
  - 9:00 to 10:10: E. Kowalski (2/4)
  - 10:10 to 10:30: Coffee break
  - 10:30 to 11:30: L. Pierce (3/3)
  - 11:30 to 14:00: Lunch
  - 14:00 to 15:00: T. Browning (2/3)
  - 15:30 to 16:30: Ph. Michel (1/4)
  - Evening: Social dinner
- **Wednesday, January 16.**
  - 9:00 to 10:10: E. Kowalski (3/4)
  - 10:10 to 10:30: Coffee break
  - 10:30 to 11:30: Ph. Michel (2/4)
  - 11:30 to 14:00: Lunch
  - 14:00 to 15:00: E. Kowalski (4/4)
  - 15:30 to 16:30: W. Sawin (1/4)
- **Thursday, January 17.**
  - 9:00 to 10:10: W. Sawin (2/4)
  - 10:10 to 10:30: Coffee break
  - 10:30 to 11:30: Ph. Michel (3/4)
  - 11:30 to 14:00: Lunch
  - 14:00 to 15:00: T. Browning (3/3)
  - 15:30 to 16:30: W. Sawin (3/4)
- **Friday, January 17.**
  - 9:00 to 10:10: W. Sawin (4/4)
  - 10:10 to 10:30: Coffee break
  - 10:30 to 11:30: Ph. Michel (4/4)
  - 11:30 to 14:00: Lunch