# The ETH Math Youth Academy

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According to the 2012 PISA survey[1], Switzerland (along with Liechtenstein) is best in Europe and 9-th in the world in terms of the general quality of mathematical education in school. The level of the students is homogeneously very high. However, opportunities for the students to go *beyond* the standard mathematical curriculum do not abound, at least in comparison to other countries (in Eastern Europe or the US, for instance).

The large–scale, long–term research project SwissMAP[2], funded by a grant from the Swiss National Foundation, includes a postdoc position at ETH Zürich with a focus on designing and implementing school outreach activities. I am holding this position since September 2015 and have launched a program aimed to provide high school students with opportunities for creative thinking and for delving deeper into exciting mathematics. The main activity of the recently–established ETH Math Youth Academy[3] consists of regular weekly "math circle" classes, taking place at ETH, on topics beyond the standard curriculum. Its design has been heavily influenced by my own participation in similar activities in a thriving environment as a high school student in Bulgaria.

The focus in the classes is on "elementary" and yet nontrivial mathematics. Namely, the treated topics belong entirely to the high school domain: no analysis or linear algebra as covered in the first years in a university. While students who continue as mathematics majors will unavoidably master the standard topics later, the development of creative, outside–of–the–box thinking is beneficial for all students, regardless of their future field of study or career path. In addition, it is easier for students to become comfortable with rigorous proofs — which are heavily emphasized in the classes — in more concrete and accessible settings. Most importantly, at least for students at this age, the *aesthetics* of mathematics can be more readily and vividly illustrated in "elementary" contexts.

The classes are in the form of mini–courses, each lasting for 3 to 6 weeks, organized by topic or technique. There are two groups of students, main and advanced, according to age and experience. Topics in the main–level course include elementary graph theory, invariants, the Pigeonhole principle. As a specific example, in a mini–course on the "Extreme principle," we discussed the theorem of Sylvester–Gallai:

*Let $S$ be a finite set of points in the plane, with the property that the line joining any two of them contains a third. Prove that all the points in $S$ lie on a line.*

*Proof.* Suppose the contrary. Among all nonzero distances from a point in $S$ to a line joining two points of $S$, take a *smallest* one, say from $P$ to $l$. Since $l$ contains at least three points of $S$, we can find two, $A$ and $B$, which are on the same side of $l$ with respect to the foot $Q$ of the perpendicular from $P$ to $l$. Say $A$ is between $Q$ and $B$. But now the distance from $A$ to the line $PB$ is smaller than the one from $P$ to $l$. $\square$

In a mini–course on elementary number theory, we start systematically and from scratch introducing divisibility and congruences, reach the theorems of Euler and Fermat, and illustrate them by many applications. A subsequent mini–course on number theory will focus on quadratic residues and Gauss's Quadratic Reciprocity Law, again with numerous applications. A Euclidean geometry mini–course develops and illustrates the properties of inversion as a transformation in the plane. Hall's Marriage theorem, along with related theorems and various applications (for example, completing Latin squares) is a topic for the advanced class. Another topic for the advanced class is unique factorization of elements in the rings of integers of certain imaginary quadratic number fields (defined and treated in a completely explicit manner), with the concrete goal of solving Diophantine equations, for example, $x^2 + 2 = y^3$ or $x^3 + y^3 = z^3$.

In this way, although many of the applications of the theory are towards problems given at various mathematical competitions, the classes do not consist of isolated "problem–solving" sessions: preparation for mathematical olympiads is a byproduct, but not the main goal by itself. Instead, I take a middle–ground between olympiad preparation and theoretical mathematical studies. In fact, I expect in the future that these mini–courses will naturally give rise to student research projects.

The target audience consists of all *interested* high school students (age 14 or above). I do not use the adjectives "gifted" or "talented," as, on the one hand, this may scare some of the modest students away, and, on the other hand, I am neither able to nor need to judge at this early stage who is gifted and who is not. Any student motivated to spend two hours a week after school doing more math is welcome to attend the classes.

Parallel to the weekly classes, I have launched an initiative for giving public talks at various high schools. These are intended to be accessible to a broad general audience and to be interesting and inspiring for all the students attending. So far, only two such public talks have taken place — one on "Mathematical induction" and one on "Mathematical games" (available online from the project's website). I expect and hope for further invitations from more schools, as this is an effective way to popularize mathematics within the broader community.

As of April 2016, there are a total of about 15 students attending regularly. I identify two main challenges for the growth of the project. First, math circles are a relatively new phenomenon in Switzerland, and the tradition is yet to be established. Second, students have a very busy and rigid school schedule, which does not allow too much pursuit of independent initiatives outside of school. However, the keen students currently attending find this type of extracurricular mathematics truly exciting, and I strive for the project to expand considerably over its second year.

## Notes

1. http://www.oecd.org/pisa/keyfindings/pisa-2012-results.htm
2. http://www.nccr-swissmap.ch
3. https://www.math.ethz.ch/eth-math-youth-academy