ETH Zürich, Basisprüfung
Analysis II D-BAUG Winter 2012
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Important notes

- Duration of the exam
  - Analysis II: 120 minutes
- Permitted aids: 10 sheets DIN A4 (= 20 pages) self-authored summary; no calculator
- All answers must be justified and the approach to the solution must be clearly illustrated. Correct, but unjustified solutions will not give any points. If you make use of a theorem from the lecture, you have to quote precisely which theorem is being used.
- All exercises carry the same weight.

$*\,*\,*$ Good luck! $*\,*\,*$

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6. Determine the plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,$$

which passes through the point $(2,1,2)$ and cuts off the smallest volume from the first octant.

7. Let $P=\big(0,y(0)\big)$ be a point on the curve C defined by the equation

$$(x+xy+y)\cos(xy)=2.$$

Compute $y(0)$ as well as the slope of the curve C at the point P.

8. Consider the region in space inside the cylinder $x^2+y^2\leq 4$, above the x-y-plane and below the plane with the equation $z=2+x$. The temperature (in degrees Centigrade) at the point (x,y,z) inside this region is given by $T(x,y,z)=2x$. Compute the average temperature of this region.

Hint: The average temperature of a region K in $\mathbb{R}^3$ with the temperature distribution $T(x,y,z)$ is given by

$$\overline{T}=\frac{1}{\mathrm{vol}(K)}\iiint_K T(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

Bitte wenden!

9. Calculate the work of the vector field $\vec{F} = (3xz - y, xz + yz, x^2 + y^2)$ along the boundary of the potato chip in Figure 1. The potato chip is the part of the surface $z = xy$, which is contained inside the cylinder $x^2 + y^2 \leq 1$, oriented as indicated in Figure 1.

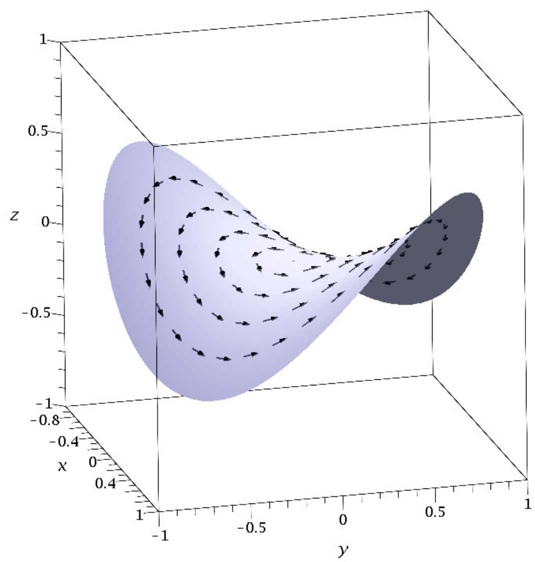

Figure 1: Task 9: The potato chip.

10. Find a solution $u(x,t)$ of the following initial-boundary value problem using separation of variables.

$$\begin{cases} \pi^2 u_t(x,t) = u_{xx}(x,t) & \text{for } 0 < x < 3 \text{ and } 0 < t \\ u(0,t) = 0 & \\ u(3,t) = 0 & \\ u(x,0) = \sin(2\pi x) & \end{cases}$$