Introduction to 3–manifolds

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Exercise 2.1. Let X and Y be oriented manifolds, and $f\colon \partial X \to \partial Y$ an orientation-reversing diffeomorphism. Show that $X \cup_f Y$ can be oriented such that $\operatorname{Int} X \subset X \cup_f Y$ and $\operatorname{Int} Y \subset X \cup_f Y$ are orientation preserving inclusions.

Exercise 2.2. Consider the relation on $\operatorname{Diff}(M)$ defined by: $f \sim g$, if there exists a diffeotopy of M from f to g. Show that this relation is an equivalence relation.

Exercise 2.3. Let $\varphi, \varphi' \in \operatorname{Diff}(\Sigma)$. Suppose there is a diffeotopy of Σ from φ to φ'. Show that the mapping tori M_φ and $M_{\varphi'}$ are diffeomorphic.

Exercise 2.4. Let $f\colon \partial M \times \mathbb{R} \to \partial M \times \mathbb{R}$ be a normalized diffeotopy of ∂M from the identity $\mathrm{id}_{\partial M}$. Show that there exists a normalized diffeotopy $F\colon M \times \mathbb{R} \to M \times \mathbb{R}$ of M with $F|_{\partial M \times \mathbb{R}} = f$.
Hint: Let f take place in a collar of ∂M and extend by the identity.

Exercise 2.5. Let $f\colon [0,1] \to [0,1]$ be a diffeomorphism with $f(0) = 0$ and $f(1) = 1$. Show that f is diffeotopic to $\mathrm{id}_{[0,1]}$.