PROJECT TOPICS

MATTHIAS NAGEL

The following projects are suggestions for topics of a bachelor thesis or a semester project under my supervision. The descriptions below are preliminary and should only give you an idea what the project is about. Very likely we adapt (or even change) them over the course of the thesis.

1. Knots in homology spheres

This project is based on the article [AR99]. Give background on homology spheres, nullhomotopic knots, Seifert surfaces and the Alexander polynomial. State the result of Freedman-Quinn that knots with trivial Alexander polynomial are topologically slice. Explain the proof of Theorem 1. Illustrate it with examples.

2. Orbifolds

Introduce orbifolds as in Chapter 13 of Thurston's notes [Thu]; see also [CHK00, Chapter 2.1]. Introduce Seifert fibred manifolds [Hat, Chapter 2.1]. Show that Seifert fibred manifolds are fibre bundles over an orbifold [Bon02, p.47]; see also [CHK00, Chapter 2.7] and [Orl72, Chapter 5.1]. Illustrate this relationship with examples.

3. Fox calculus and the cellular chain complex

This project is roughly based on the chapter [Tur01, Chapter II.16]. Construct a CW-structure on a knot exterior X_K. Introduce the (universal) cellular chain complex $C_\bullet(X_K;\mathbb{Z}[\pi_1(X_K)])$, which is the cellular chain complex of the universal cover $\widetilde{X}_K$ and the chain modules are modules with a $\pi_1(X_K)$–action. Compute the boundary maps ∂_2 and ∂_1 explicitly. Use the chain complex $C_\bullet(X_K;\mathbb{Z}[t^{\pm 1}])$ to get a presentation matrix for $H_1(X_K;\mathbb{Z}[t^{\pm 1}])$, and compute the Alexander polynomial Δ_K of K; see also [Rol90, Chapter 8], [Lic97, Chapter 6] and [Lic97, pp. 116-119]. Compute examples.

References

[AR99] David Austin and Dale Rolfsen. Homotopy of knots and the Alexander polynomial. *Canad. Math. Bull.*, 42(3):257–262, 1999.

[Bon02] Francis Bonahon. Geometric structures on 3-manifolds. In *Handbook of geometric topology*, pages 93–164. North-Holland, Amsterdam, 2002.

[CHK00] Daryl Cooper, Craig D. Hodgson, and Steven P. Kerckhoff. *Three-dimensional orbifolds and cone-manifolds*, volume 5 of *MSJ Memoirs*. Mathematical Society of Japan, Tokyo, 2000. With a postface by Sadayoshi Kojima.

[Hat] A. Hatcher. Notes on basic 3-manifold topology. available under https://pi.math.cornell.edu/~hatcher/3M/3Mdownloads.html.

[Lic97] W. B. Raymond Lickorish. *An introduction to knot theory*, volume 175 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1997.

[Orl72] Peter Orlik. *Seifert manifolds*. Lecture Notes in Mathematics, Vol. 291. Springer-Verlag, Berlin-New York, 1972.

[Rol90] Dale Rolfsen. *Knots and links*, volume 7 of *Mathematics Lecture Series*. Publish or Perish, Inc., Houston, TX, 1990. Corrected reprint of the 1976 original.

[Thu] W. Thurston. Geometry and topology of three-manifolds. available under http://library.msri.org/books/gt3m/.

[Tur01] Vladimir Turaev. *Introduction to combinatorial torsions*. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 2001. Notes taken by Felix Schlenk.

Department of Mathematics, ETH Zurich, Switzerland
Email address: matthias.nagel@math.ethz.ch