Numerical analysis in Zurich – 50 years ago*

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Surely, applied mathematics originated in ancient times and slowly matured through the centuries, but it started to blossom colorfully only when electronic computers became available in the late 1940s and early 1950s. This was the gold miners' time of computer builders and numerical analysts. The venue was not the far west of the United States, but rather some places in its eastern part, such as Boston, Princeton, Philadelphia, and New York, and also places in Europe, most notably, Manchester, Amsterdam, and Zurich. Only long after these projects had begun it became known that the electronic computer had been invented earlier by clever individuals: 1937–1939 by John V. Atanasoff and his graduate student Clifford Berry at Iowa State College, Ames, Iowa, and, independently, 1935–1941 by Konrad Zuse in Berlin. We want to recall here some of the Swiss contributions. We focus on those in numerical analysis and scientific computing, but we will also touch computers, computer languages, and compilers.

Responsible for establishing (electronic) scientific computing in Switzerland was primarily Eduard Stiefel (21.4.1909–25.11.1978): he took the initiative, raised the money, hired the right people, directed the projects, and, last but not least, made his own lasting contributions to pure and applied mathematics. Stiefel got his Dr. sc. math. from ETH in 1935 with a dissertation on "Richtungsfelder und Fernparallelismus in n-dimensionalen Mannigfaltigkeiten" written under the famous Heinz Hopf. It culminated in the introduction of the Stiefel(–Whitney) classes, certain characteristic classes associated with real vector bundles. Eduard Stiefel left further traces in pure mathematics with the Stiefel manifold and, later, in 1942, with the Stiefel diagram related to Lie groups and crystallographic groups. Decades later he would time and again capitalize on this knowledge by applying it to various problems in applied mathematics.

In WW II, Stiefel spent much time in the army, where he reached the rank of a colonel, commanding the artillery weather services. That must have whetted his appetite for numerical computations. Moreover, soon after the war he must have heard of projects in other countries to build electronic computers, and he quickly started an initiative for establishing an Institute for Applied Mathematics at ETH. Its prime aim was to build an electronic computer – there were none for sale then – and to find out how to use it.

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There were no operating systems yet, no compilers, no programming languages, and little algorithmic thinking. As it turned out, Stiefel's institute, which was founded officially on January 1, 1948, became one of the few places worldwide that made fundamental contributions to all these areas within 10 years.

Figure 1. Eduard Stiefel, the chief, who had an interest in fishing.

After establishing the institute, Stiefel soon hired two assistants: a young mathematician named Heinz Rutishauser (30.1.1918–10.11.1970) and a young electrical engineer named Ambros Speiser (13.11.1922–10.5.2003). Rutishauser had been working as a high-school teacher and was about to finish his dissertation on a topic in complex analysis. Speiser was just about to get his diploma. Next, Stiefel went for five months (from October 18, 1948 to March 12, 1949) on a fact finding mission to Amsterdam and the United States. He visited several computer building projects and some colleagues who worked on algorithms for scientific computing. He also organized for each of his two assistants to spend the whole year of 1949 in the US. Each of them could stay half of the time with Howard Aiken at Harvard University and the other half with John von Neumann at Princeton. What a generous boss! And what a clever move for know-how acquisition! Fostering these two young researchers payed off well – as it generally does.

When Stiefel returned in Spring 1949 he had to anticipate that designing and constructing an electronic computer in Zurich would take several years, and during this time work on numerical algorithms would be limited by the impossibility of testing them. He was more than happy to learn one day that Konrad Zuse (22.6.1910–18.12.1995), who lived in the small

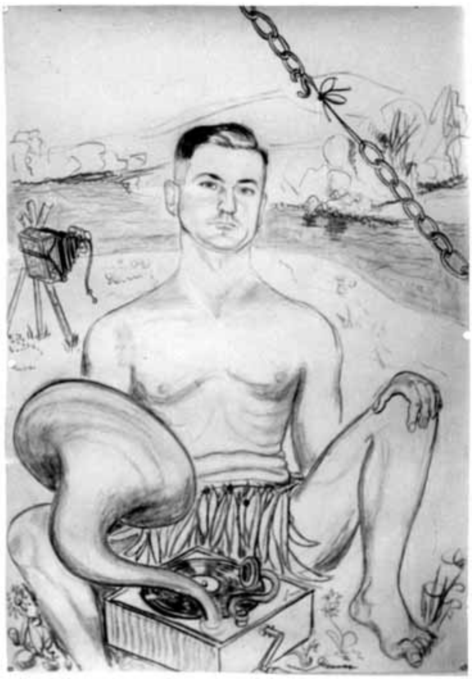

Figure 2. Heinz Rutishauser, fond of music, shown below a broken chain, in German *Kettenbruch*, i.e., continued fraction.

Bavarian village Hopferau, offered to lease him an operational digital electromechanical computer. Konrad Zuse was a real inventor. In 1935–1945 he designed a series of four digital computers with binary floating-point representation in his parents' apartment in Berlin. While the Z1 was still fully mechanical, the Z2 of 1940 had already an arithmetic unit assembled from electromagnetic relays. It was, however, error-prone, but useful as a prototype to get support for his next project. The Z3 was completed in 1941 and was a fully operational programmable computer based on the relay technology: 600 relays were used for the CPU, 1400 for the memory. A replica is now on display in the German Museum in Munich. Finally, in

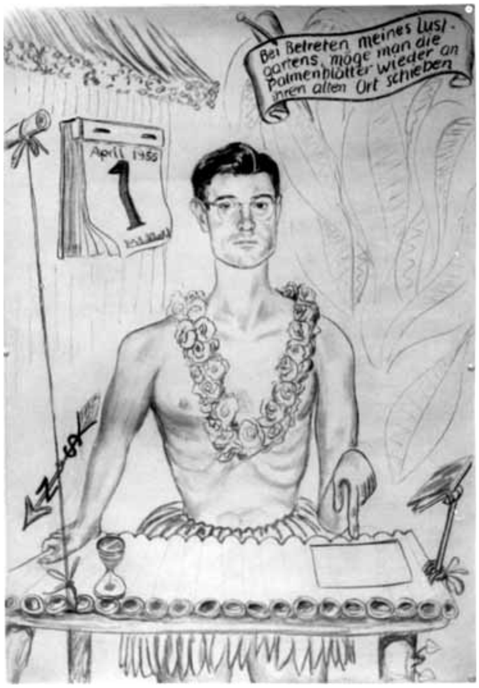

Figure 3. Ambros Speiser, who was always trying to maintain the site in order.

1942–1945, Zuse built the more powerful Z4. In the last month of the war, he was able to flee the capital with his Z4 packed up in boxes and to retreat to the secluded village of Hinterstein, close to the Austrian border. Once the end-of-war turmoil had cooled down, Zuse moved to the better accessible Hopferau, where, in 1946–1949, he completed the Z4 in the former flour storage room of a bakery. After inspecting the machine, Stiefel leased the Z4 for CHF 30'000 for five years. It was installed at ETH in August 1950, often ran day and night, and proved very reliable, except for the memory (64 32-bit numbers), which was still mechanical. The technical details have been reported in many newspaper and journal articles, see, e.g., [19], [16], [5], and the references listed in there. Some new features were added in Zurich, e.g., the treatment of conditional branches.

Despite the Z4, designing a more powerful electronic computer to be called ERMETH (Elektronische Rechenmaschine der ETH) was still the main target of Stiefel's quickly growing institute. Responsible for this task was Ambros Speiser, in the meantime the technical director of a group of five engineers and three mechanics. But reportedly, Rutishauser's ingenious views also had a great influence on the design. Among the traces of their work are the two reports [15], [10], the latter being Rutishauser's habilitation thesis describing what we would call now a compiler. Ultimately, the ERMETH started running in July 1956 and remained operational till 1963. Its most impressive part was a large magnetic drum that could store 10′000 floating-point numbers in a 16-decimal representation with an 11-decimal

Figure 4. The South Seas island of ERMETHIA.

mantissa. For further details on the ERMETH see [9] and the references given there.

Running the Z4 and at the same time building the ERMETH was a lot of hard work for the team of the institute, work often carried out day and night. The excellent team spirit is reflected in our illustrations that show the mystic South Seas island of ERMETHIA and the four most prominent team members as its inhabitants. The drawings were sketched by Alfred Schai (21.4.1928–28.5.2009) in coal on large sheets of paper around 1956, but only photographs survived the years. Schai was an electrical engineer of the team, who later, 1964–1989, was director of the well equipped Computer Center of ETH Zurich.

The availability of the Z4 allowed Stiefel and his collaborators to explore numerical methods on it. They also attacked real applications from civil engineering (dams, plates), quantum chemistry (eigenvalues), and airplane design (deformation of wings). Most of the textbook methods of this area originated from the time when "computers" were still human beings. Some of these methods were suitable for programmable electronic computers, others proved inappropriate or inefficient. The strong limitations regarding memory and program complexity posed severe difficulties and

Figure 5. Hans Schlaeppi, responsible for the design and construction of the magnetic drum memory.

required special attention. For example, it was important to take advantage of the sparsity and structure of a matrix, and it was easy to do that for the matrices resulting from the finite-difference method if one applied iterative methods for solving linear systems.

On his first trip to the USA, Stiefel met Garrett Birkhoff and his student David Young, who was working on the theory of a clever improvement of the Gauss–Seidel method. Young's successive overrelaxation (SOR) method proved to be very efficient for many symmetric positive definite (spd) problems (and a few others). Stiefel also knew of a number of other, competitive, "relaxation" methods that were equally simple to apply and did not use a matrix splitting, but – like SOR – required choosing certain parameters. The best such method is known as Chebyshev iteration as it makes use of Chebyshev polynomials. But Stiefel was looking for a better method that automatically adapts to each matrix and to the initial approximation of the solution. He discovered it in 1951: an iterative method for spd problems that delivers in every step an optimal approximation of the solution and constructs it with an update process based on simple 2-term recurrences.

He called it the n-step method, as it delivers the exact solution of a linear n-by-n system in at most n steps. He first mentioned it in a paper submitted in July 1951, in which he reviewed various approaches to the iterative solution of linear systems [17].

In the same month he travelled to the United States for a second, even longer visit (from July 1951 to February 1952), this time to the Institute for Numerical Analysis (INA) of the National Bureau of Standards, which was located at UCLA in Los Angeles. He was invited by Olga Taussky, who knew him from his earlier work in pure mathematics. His visit was scheduled to include the Symposium on Simultaneous Linear Equations and the Determination of Eigenvalues, held at the INA on August 23–25, 1951, and it was there that he found out that Magnus Hestenes of the INA (in collaboration with others of the same institute) had discovered the same method too. Hestenes, who called it the conjugate gradient (CG) method, had published an internal report on it also in July [6]. Stiefel's long visit gave a perfect chance for a deeper joint investigation. The resulting 28 page two-column paper [7] is one of the most influential papers in numerical analysis. Not only did it make the very effective CG method known to a larger audience, but it also fully explored the method, from related mathematical theories (orthogonal polynomials, continued fractions) to details of implementation (including a very clever stopping criterion and experiments on round-off effects), from possible generalizations (conjugate-direction methods) to particular applications. In the subsequent fifty years, innumerable publications discussed variations and generalizations of the CG method, which is the role model of what we call now a Krylov subspace method. (The Chebyshev iteration is also a Krylov subspace method, but in contrast to CG it is neither parameter-free nor optimal.) In Zurich, Stiefel had his student Urs Hochstrasser code the method on the Z4, and they were able to solve linear systems with up to 106 unknowns – quite impressive for a memory of 64 numbers! In fact, intermediate results were stored externally by punching holes in old movie films. For further details on the history of the CG method see [2] and the more recently provided historical documents on the Web [20], which include Hochstrasser's presentation at the Latsis Symposium 2002 commemorating the publication of the Hestenes–Stiefel paper 50 years before.

While being occupied with the design of the ERMETH and fundamental questions regarding computer programming, Stiefel's collaborator Rutishauser was also engaged in developing numerical algorithms. He studied in particular the seminal paper of Lanczos [8] on solving eigenvalue problems (with a Krylov subspace method closely related to CG) and, on suggestion of Stiefel, he approached the problem of finding all the eigenvalues of a matrix from a sequence of so-called moments $c_k := \tilde{y}_0^\top A^k y_0$ $(k = 0, 1, \dots)$, where $\tilde{y}_0$ and y_0 are suitably chosen initial vectors. Finding eigenvalues is a much more challenging problem than solving linear systems of equations, and the methods of the time were quite limited. Many were based on constructing the characteristic polynomial in one way or another, an approach that was shortly after discarded for many reasons. In 1953, improving on ideas of D. Bernoulli, Hadamard, and Aitken, Rutishauser came up with a completely new method, the quotient-difference or qd algorithm. He discovered that it had connections to Lanczos' work, to the CG method, to (formal) orthogonal polynomials and to continued fractions – that is, to the same circle of ideas Hestenes and Stiefel had encountered. He noted that the original idea of computing the eigenvalues from the moments was a bad one due to ill-conditioning, but that combining the Lanczos algorithm with a "progressive" version of the qd algorithm gave much better results. Moreover, one day in 1954 he realized that one sweep of the progressive qd algorithm can be mimicked by factoring (if possible) a tridiagonal matrix into a unit lower bidiagonal matrix L times an upper bidiagonal matrix R, and then multiplying the factors in reverse order to get another tridiagonal matrix, which is similar to the first one. When such LR steps were repeated *ad infinitum* the convergence of the qd algorithm was reflected in the convergence of the above mentioned bidiagonal matrices L and R to the unit matrix and a bidiagonal matrix containing the eigenvalues, respectively. This was the birth of the LR algorithm, which, as was easy to see, also worked for full matrices. Rutishauser also discovered that by suitably shifting the spectrum of the matrix the qd and LR algorithms could attain quadratic or even cubic convergence.

Over the years, he found other variants of the qd and LR algorithms and also further applications for them. About 20 of his publications are somehow related to qd or LR. Best known are [11] and [12]. Readers interested in the effects of finite-precision arithmetic should consult the appendices of [14]. Yet, he missed finding the most important generalization: it was J. G. F. Francis who, soon after the publication of [12], submitted a first version of a paper, where the idea of factorizing a matrix and assembling the factors in reverse order for obtaining a matrix similar to the original one was used with a QR factorization instead of an LR (or, in English notation, LU) factorization. Francis' QR algorithm became the standard tool for eigenvalue computations. Only recently has it been challenged by new methods, including the so-called differential qd algorithm, which was rediscovered and perfected by Fernando and Parlett. For further details on the history of qd and LR see [4], and for the history of QR see [3].

The CG, qd, and LR methods are just the three most important topics in numerical analysis that were treated in Zurich in the 1950s and 1960s. And Rutishauser is just the most famous out of a long list of Stiefel's early collaborators, many of whom became professors in numerical analysis (Rutishauser at ETH Zurich; Jean Descloux at EPF Lausanne; Hans-Rudolf Schwarz at the University of Zurich), computer science (Peter Läuchli and Carl August Zehnder at ETH Zurich), and related areas (Max Engeli at ETH Zurich for computer-aided design). In 1955, Ambros Speiser became – at

Figure 6. Peter Läuchli, after 1955 responsible for the ERMETH's logic.

age 33 – the founding director of the IBM Research Laboratory near Zurich (one of only three worldwide), and in 1966 the founding director of the BBC (later ABB) Research Laboratory near Baden, Switzerland. Moreover, Stiefel had yet other students with careers in pure and applied mathematics.

Besides Stiefel, another important Swiss figure with an interest in numerical analysis was Alexander Ostrowski (25.9.1893–20.11.1986). Born in Kiew he was a professor in Basel from 1927 till his retirement in 1958. His work spans nearly all areas of mathematics, but his mostly theoretical work in numerical analysis alone is quite impressive; and he also had a number of well-known students.[1]

The enthusiasm for computers and applied mathematics that arose in Switzerland in the 1950s, stirred by the activities of the Institute, actually produced more students than the market could absorb. Conse-

[1] See “Alexander M. Ostrowski (1893–1986): His life, work, and students” by Walter Gautschi, pp. 257–278 in this volume.

quently a number of young Swiss applied mathematicians emigrated to the USA. Many of them, but not all, returned after a few years. Peter Henrici (13.9.1923–13.3.1987), who got his Dr. sc. math. (as he liked to stress) "only formally" under Stiefel, Urs Hochstrasser (b. 1926), student of Stiefel, and Walter Gautschi (b. 1927), a student of Ostrowski, all started with positions at the American University in Washington, D.C., that allowed them to work for the National Bureau of Standards. There, Gautschi and Hochstrasser contributed chapters to the ubiquitous Handbook of Mathematical Functions of Abramowitz and Stegun. Henrici soon became a faculty member at UCLA and returned to ETH Zurich in 1962. Hochstrasser, who already had had a fellowship at the INA at UCLA while Stiefel and Hestenes were working out their CG paper, was a professor and computer center director in Kansas. In 1958 he became the first Swiss science attaché in Washington and Ottawa. Later, 1969–1989 he was the Swiss top official for Education and Science, director of the *Bundesamt für Bildung und Wissenschaft* (previously *Abteilung für Wissenschaft und Forschung*), and in this position he kept teaching numerical analysis courses at the University of Bern. Walter Gautschi is one of those who did not return: as is well known, he ended up in Purdue, where he has been on the faculty for some 40 years. Unfortunately, two other Swiss emigrants with promising careers died early. Werner Gautschi (1927–1959), Walter's twin brother and also a student of Ostrowski, took positions in Princeton, Berkeley, Ohio State, and Indiana before his untimely death [1]. Hans Jakob Maehly (1920–1961), who got his PhD in physics at ETH in 1951 under the famous Paul Scherrer for a dissertation on eigenvalue computations, was at Princeton and Syracuse University and shortly at the Argonne National Lab before his premature death on Nov. 16, 1961.

Before we come to an end, we need to mention yet another area of early work in the Institute for Applied Mathematics: the intense participation in the international collaboration that created the seminal programming language ALGOL 60. Again, it was Heinz Rutishauser who, along with his colleagues Friedrich L. Bauer and Klaus Samelson in Munich, got strongly engaged in the definition and use of this language. Hans-Rudolf Schwarz wrote a compiler for its use on the ERMETH. The idea of creating a programming language to be used world-wide for numerical computations was fascinating. As mentioned before, Rutishauser had already developed ideas about a programming language and compilation in [10]. He contributed many ideas to ALGOL 60, wrote a complete textbook for programming in ALGOL 60 with many beautiful examples of numerical algorithms [13], and, together with his colleagues in Zurich and Munich, contributed many perfectly coded numerical procedures to the Handbook project [18]. As he once told us in a lecture, not all of his ideas had been accepted however:

for example, he wanted to use a wildcard notation to make it possible to specify a column or a row of a matrix as actual values for a vector-valued variable of a function or procedure, as we are used to do it in MATLAB today. Unfortunately, ALGOL 60 did not find the world-wide acceptance it deserved, but most of the ideas behind it reappeared later in modern computer languages.

Rutishauser's enormous work in computer science and numerical analysis becomes even more astonishing when one knows that as early as 1955 he had heart problems, which ultimately lead to his early death at age 52 in 1970.

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