

TATE COHOMOLOGY FOR ARBITRARY GROUPS VIA SATELLITES

GUIDO MISLIN

ETH Mathematik, Zürich, Switzerland, and Department
of Mathematics, Ohio State University, Columbus, Ohio

ABSTRACT. We define cohomology groups $\hat{H}^n(G; M)$, $n \in \mathbb{Z}$, for an arbitrary group G and G -module M , using the concept of satellites. These cohomology groups generalize the Farrell-Tate groups for groups of finite virtual cohomological dimension and form a connected sequence of functors, characterized by a natural universal property.

The classical Tate cohomology groups of finite groups have been generalized to larger classes of groups by several authors ([BC], [F], [GG], [I]). The definition of Benson and Carlson in [BC] makes sense for an arbitrary group, but no formal properties are discussed there. We propose here a different definition for Tate cohomology groups for an arbitrary group G and G -module M , which takes the form

$$\hat{H}^n(G; M) = \varinjlim_{j \geq 0} S^{-j} H^{n+j}(G; M)$$

with $S^{-j} H^{n+j}(G; ?)$ denoting the j -th left satellite of the functor $H^{n+j}(G; ?)$. These general Tate groups are shown to agree, if applicable, with the ones obtained by the various generalizations mentioned above. The family $\hat{H}^\bullet = \{\hat{H}^n(G; ?); n \in \mathbb{Z}\}$ forms a connected sequence of functors and is as such characterized by a natural universal property, which identifies it with what we call the *completion with respect to projective modules*, or short, the *P-completion* of ordinary cohomology (cf. section 3).

In the first section we recall, to fix our terminology and notation, some basic facts on satellites. Section two is devoted to an axiomatic description of the *P-completion* of a cohomological functor, leading to our definition of the general Tate groups. In section three we compare the definition to various other ones and discuss a few examples. Section four contains a detailed comparison with the definition proposed by Benson and Carlson.

We thank Karl Gruenberg for enlightening discussions concerning the concept of projective completion for connected sequences of functors.

1991 *Mathematics Subject Classification*. Primary 18Gxx, 55Uxx; Secondary 20Jxx.

Key words and phrases. cohomology functors, Tate cohomology.

Section 1 : Satellites

The basic references here are [CE] and [HS]. We will quickly review the facts of which we make use. Let Λ be an associative ring with 1 and M a (left) Λ -module. We write FM for the free Λ -module on the underlying set of M , and ΩM for the kernel of the obvious map $FM \rightarrow M$. If T denotes an additive functor from Λ -modules to abelian groups, then

$$S^{-1}T(M) = \ker(T(\Omega M) \rightarrow T(FM))$$

defines a new additive functor $S^{-1}T$, the left satellite of T . For $n \geq 1$ one defines inductively $S^{-n}T = S^{-1}(S^{-n+1}T)$ and $\Omega^n M = \Omega(\Omega^{n-1}M)$ with the convention that $S^0T = T$ and $\Omega^0 M = M$, respectively. Each short exact sequence of Λ -modules $A' \rightarrow A \rightarrow A''$ gives rise to a connecting homomorphism $S^{-n}T(A'') \rightarrow S^{-n+1}T(A')$, $n > 0$, in such a way that in the long sequence

$$\cdots \rightarrow S^{-n}TA' \rightarrow S^{-n}TA \rightarrow S^{-n}TA'' \rightarrow S^{-n+1}TA' \rightarrow \cdots \rightarrow TA'',$$

the composition of any two consecutive homomorphisms is zero. Thus the family $S^{\leq 0}T = \{S^{-n}T; n \geq 0\}$ forms a connected sequence of functors. Obviously, $S^{-1}T(P) = 0$ for all projective modules P , because $\Omega P \rightarrow FP$ is a split monomorphism. As a result, one has for general M and $n > k \geq 0$ natural isomorphisms

$$(1.1) \quad S^{-n}TM \cong S^{-n+k}T\Omega^k M.$$

A connected sequence of functors $V^{\leq 0} = \{V^{-n}; n \geq 0\}$ is called of cohomological type, if the long sequence

$$\cdots \rightarrow V^{-n}A' \rightarrow V^{-n}A \rightarrow V^{-n}A'' \rightarrow V^{-n+1}A' \rightarrow \cdots \rightarrow V^0A'',$$

associated with any short exact sequence $A' \rightarrow A \rightarrow A''$, is exact; in the terminology of [GG] such a $V^{\leq 0}$ is called a $(-\infty, 0)$ -cohomological functor. For instance, if T is additive and half exact, then $S^{\leq 0}T$ is of cohomological type (cf.[CE]). Any natural transformation $\phi : U \rightarrow V$ of additive functors extends uniquely to a morphism $\phi^{\leq 0} : S^{\leq 0}U \rightarrow S^{\leq 0}V$ of connected sequences of functors. More generally, if $V^{\leq 0}$ is any connected sequence of (additive) functors, then any natural transformation $\psi : V^0 \rightarrow V^0$ extends uniquely to $V^{\leq 0} \rightarrow S^{\leq 0}V$; in particular, by taking for ψ the the identity of V^0 , one obtains a morphism $V^{\leq 0} \rightarrow S^{\leq 0}V$ which we call the canonical one. The connected sequence of left satellites of a half exact functor can be characterized as follows (cf.[CE], III 5.2).

Theorem 1.2. *Let $U^{\leq 0}$ and $V^{\leq 0}$ denote connected sequences of (additive) functors and $\phi^0 : U^0 \rightarrow V^0$ a natural transformation. If $V^{\leq 0}$ is of cohomological type and satisfies $V^{-n}(P) = 0$ for all $n > 0$ and all projective P , then the following holds:*

- (1) ϕ^0 extends uniquely to $\phi^{\leq 0} : U^{\leq 0} \rightarrow V^{\leq 0}$ and $\phi^{\leq 0}$ factors uniquely through the canonical morphism $U^{\leq 0} \rightarrow S^{\leq 0}U^0$
- (2) if U^0 is half exact and ϕ^0 is an equivalence then the induced morphism $S^{\leq 0}U^0 \rightarrow V^{\leq 0}$ is an equivalence.

Section 2 : P-complete functors

We will follow the terminology of [GG] and call a connected sequence of additive functors $T^\bullet = \{T^n; n \in \mathbb{Z}\}$ a $(-\infty, +\infty)$ -cohomological functor, if the long sequence

$$\cdots \rightarrow T^n A' \rightarrow T^n A \rightarrow T^n A'' \rightarrow T^{n+1} A' \rightarrow \cdots$$

associated with any short exact sequence $A' \rightarrow A \rightarrow A''$ of Λ -modules is exact. A typical example is given by ordinary cohomology $H^\bullet = \{H^n(G; ?); n \in \mathbb{Z}\}$, with the convention that $H^n(G; ?) = 0$ for $n < 0$.

Definition 2.1. A $(-\infty, +\infty)$ -cohomological functor $T^\bullet = \{T^n; n \in \mathbb{Z}\}$ is called *P-complete*, if $T^n(P) = 0$ for every n and every projective module P . A morphism $U^\bullet \rightarrow V^\bullet$ of $(-\infty, +\infty)$ -cohomological functors is called a *P-completion*, if V^\bullet is *P-complete* and if every morphism $U^\bullet \rightarrow W^\bullet$ into a *P-complete* cohomological functor W^\bullet factors uniquely through $U^\bullet \rightarrow V^\bullet$.

If G is a finite group, then the classical Tate groups $\hat{H}^\bullet = \{\hat{H}^n(G; ?); n \in \mathbb{Z}\}$ form a *P-complete* cohomological functor and the natural morphism $H^\bullet \rightarrow \hat{H}^\bullet$ is a *P-completion* (see also 3.1). More generally, if the $(-\infty, +\infty)$ -cohomological functor U^\bullet admits a “terminal completion” $U^\bullet \rightarrow V^\bullet$ in the sense of [GG] then it follows that $U^\bullet \rightarrow V^\bullet$ is a *P-completion* (we will discuss this in section 3). Since not every U^\bullet admits a “terminal completion”, one can, in view of the following theorem, think of the *P-completion* as a natural generalization of the “terminal completion” of [GG].

Theorem 2.2. Every $(-\infty, +\infty)$ -cohomological functor $T^\bullet = \{T^n; n \in \mathbb{Z}\}$ admits a unique *P-completion* $\tau^\bullet : T^\bullet \rightarrow \hat{T}^\bullet$.

Proof. For every $n \in \mathbb{Z}$ we can form the $(-\infty, n)$ -cohomological functor $S^{\leq 0}T^n$ which extends to a $(-\infty, +\infty)$ -cohomological functor $T^\bullet \langle n \rangle$ by putting

$$(2.3) \quad T^j \langle n \rangle = \begin{cases} S^{j-n}T^n, & \text{if } j < n \\ T^j, & \text{if } j \geq n. \end{cases}$$

The identity transformation $T^n \rightarrow T^n$ extends uniquely to $T^{\leq n} \rightarrow S^{\leq 0}T^n$, and we extend it further to $\tau_n^\bullet : T^\bullet \rightarrow T^\bullet \langle n \rangle$ by putting $\tau_n^j = Id_{T^j}$ for $j > n$. Similarly, for any $m \geq n$ the identity $T^m \rightarrow T^m$ extends uniquely to a morphism $\tau_{n,m}^\bullet : T^\bullet \langle n \rangle \rightarrow T^\bullet \langle m \rangle$ satisfying $\tau_{n,m}^j = Id_{T^j}$ for each $j \geq m$. We define now

$$\hat{T}^\bullet = \varinjlim \{T^\bullet \langle n \rangle; \tau_{n,m}^\bullet\}.$$

Because $\tau_{n,m}^\bullet \circ \tau_n^\bullet = \tau_m^\bullet$ for $m \geq n$, we obtain a natural morphism

$$\tau^\bullet = \varinjlim \tau_n^\bullet : T^\bullet \rightarrow \hat{T}^\bullet.$$

The exactness of \varinjlim implies that \hat{T}^\bullet is a $(-\infty, +\infty)$ -cohomological functor. By our definition, we have for any M

$$\hat{T}^j(M) = \varinjlim_{k \geq 0} S^{-k}T^{j+k}(M)$$

so that for P projective $\hat{T}^j(P) = 0$ for any j , because $S^{-k}T^{j+k}(P) = 0$ for $k > 0$. Thus \hat{T}^\bullet is P -complete. For the universal property of τ^\bullet consider $T^\bullet \rightarrow V^\bullet$ with V^\bullet a P -complete $(-\infty, +\infty)$ -cohomological functor. Then each $T^n \rightarrow V^n$ extends uniquely to $S^{\leq 0}T^n \rightarrow S^{\leq 0}V^n$, and $S^{\leq 0}V^n \cong V^{\leq 0}$ by (1.2). In this way we obtain for each n a unique morphism $T^\bullet\langle n \rangle \rightarrow V^\bullet$ factoring $T^\bullet \rightarrow V^\bullet$ as $T^\bullet \rightarrow T^\bullet\langle n \rangle \rightarrow V^\bullet$. As a result, $T^\bullet \rightarrow V^\bullet$ factors uniquely through τ^\bullet . The uniqueness of the P -completion is a consequence of its definition.

The following two lemmas are useful for computations.

Lemma 2.4. *If T^\bullet is a $(-\infty, +\infty)$ -cohomological functor and $n_0 \in \mathbb{Z}$ satisfies $T^n(P) = 0$ for all $n \geq n_0$ and all P projective, then $\tau^n(M) : T^n(M) \rightarrow \hat{T}^n(M)$ is an isomorphism for all $n \geq n_0$ and \hat{T}^\bullet is naturally equivalent to $T^\bullet\langle n \rangle$.*

Proof. Because $T^m(P) = 0$ for P projective and $m \geq n_0$ we have, similarly as in (1.1), for all $n \geq n_0$ and all $k \geq 0$ natural isomorphisms $S^{-k}T^{n+k}(M) \cong T^{n+k}(\Omega^k M)$, and also $T^{n+k}(\Omega^k M) \cong T^n(M)$. As a result,

$$\hat{T}^n(M) = \lim_{\substack{\longrightarrow \\ k \geq 0}} S^{-k}T^{n+k}(M) \cong T^n(M).$$

For $n \geq n_0$, $T^\bullet\langle n \rangle$ is P -complete and thus $T^\bullet \rightarrow T^\bullet\langle n \rangle$ induces $\hat{T}^\bullet \rightarrow T^\bullet\langle n \rangle$, which is inverse to the natural map $T^\bullet\langle n \rangle \rightarrow \hat{T}^\bullet$.

Lemma 2.5. *If $\phi^\bullet : T^\bullet \rightarrow V^\bullet$ is a morphism of $(-\infty, +\infty)$ -cohomological functors with V^\bullet P -complete and if $\phi^n : T^n \rightarrow V^n$ is an equivalence for $n \geq n_0$, then the induced morphism $\hat{T}^\bullet \rightarrow V^\bullet$ is an equivalence.*

Proof. We apply a ‘‘dimension shifting’’ argument as follows. Since \hat{T}^\bullet and V^\bullet are P -complete, they satisfy for any $k \in \mathbb{Z}$ and any M

$$\hat{T}^k(M) \cong \hat{T}^{k+1}(\Omega M), V^k(M) \cong V^{k+1}(\Omega M).$$

Thus it suffices to show that $\hat{T}^k \rightarrow V^k$ is an equivalence for $k \geq n_0$. As $T^k(P) = 0$ for $k \geq n_0$, we know from (2.4) that $T^k \cong \hat{T}^k$ for $k \geq n_0$, and the conclusion follows, since ϕ^k is an equivalence for $k \geq n_0$.

Section 3 : Examples

Let G be an arbitrary group and consider the $(-\infty, +\infty)$ -cohomological functor $H^\bullet = \{H^n(G; ?); n \in \mathbb{Z}\}$ given by ordinary cohomology with $H^n(G; ?) = 0$ for $n < 0$. It has a P -completion $H^\bullet \rightarrow \hat{H}^\bullet$ and we call the associated groups $\hat{H}^n(G; M)$ the n -th Tate cohomology groups of G with coefficients in the G -module M . Thus for any $n \in \mathbb{Z}$,

$$\hat{H}^n(G; M) = \lim_{\substack{\longrightarrow \\ k \geq 0}} S^{-k}H^{k+n}(G; M),$$

and the morphism $H^\bullet \rightarrow \hat{H}^\bullet$ into Tate cohomology is universal with respect to morphisms $H^\bullet \rightarrow V^\bullet$ into P -complete $(-\infty, +\infty)$ -cohomological functors V^\bullet . These Tate groups generalize the classical Tate groups for finite groups. More generally, the following holds.

Lemma 3.1. *Let G denote a group of finite virtual cohomological dimension. Then the P -completion of $H^\bullet = \{H^n(G; ?); n \in \mathbb{Z}\}$ is naturally equivalent to Farrell cohomology.*

Proof. Consider the natural morphism $H^\bullet \rightarrow F^\bullet$ from ordinary cohomology to Farrell cohomology F^\bullet . Since F^\bullet is a P -complete $(-\infty, +\infty)$ -cohomological functor and since $H^n \rightarrow F^n$ is an equivalence for $n > vcd(G)$, we infer from (2.5) that $\hat{H}^\bullet \cong F^\bullet$.

If G is a group such that for some integer n_0 one has $H^n(G; P) = 0$ for all $n \geq n_0$ and all P projective, then $H^\bullet = \{H^n(G; ?); n \in \mathbb{Z}\}$ admits a “terminal completion” T^\bullet in the sense of [GG] which is given by a morphism $H^\bullet \rightarrow T^\bullet$ such that $H^n(G; ?) \rightarrow T^n$ is an equivalence for $n \geq n_0$, and T^\bullet is actually given by $H^\bullet \langle n_0 \rangle$ (loc. cit.); since $H^\bullet \langle n_0 \rangle$ is P -complete, the natural morphism $H^\bullet \langle n_0 \rangle \rightarrow \hat{H}^\bullet$ is an equivalence by (2.5), and the “terminal completion” T^\bullet is therefore naturally equivalent to the P -completion \hat{H}^\bullet .

It is clear from the definition of the P -completion that for an arbitrary group G the following two conditions are equivalent:

- (i) $H^\bullet(G; ?) \rightarrow \hat{H}^\bullet(G; ?)$ is an equivalence
- (ii) $H^n(G; P) = 0$ for all n and all projective P .

There are indeed examples of groups satisfying these conditions. In [BG] a finitely presented group of type FP_∞ satisfying (ii) is described. It is easy to check that an infinite free abelian group of countable rank satisfies the condition (ii) too. The following theorem provides further examples, which might help to understand the cohomology of $Gl_n(\overline{\mathbb{Q}})$ with \mathbb{F}_p -coefficients, a problem which is closely related to a conjecture of Friedlander and Milnor [M].

Theorem 3.2. *Let $K \subset \overline{\mathbb{Q}}$ be a subfield of the algebraic closure of the rational numbers and let $j \geq 1$. Then the P -completion*

$$H^\bullet(Gl_j(K); ?) \rightarrow \hat{H}^\bullet(Gl_j(K); ?)$$

is an equivalence.

Proof. We can write K as a countable union $\bigcup \mathcal{O}_{S_i}(K_i)$ with each $K_i \subset K$ a number field and $\mathcal{O}_{S_i}(K_i) \subset K_i$ the ring of S_i -integers, S_i a finite set of primes of K_i , and $i \in \mathbb{N}$. Without loss of generality we may assume that $\mathcal{O}_{S_i}(K_i) \subset \mathcal{O}_{S_{i+1}}(K_{i+1})$ for all i . Note that $vcd(Gl_j(\mathcal{O}_{S_i}(K_i))) = n_i < \infty$ and $\lim_{i \rightarrow \infty} n_i = \infty$. Let P be a projective $G = Gl_j(K)$ -module. We obtain then a short exact sequence

$$(3.3) \quad \varprojlim_{i \in \mathbb{N}} H^{n-1}(Gl_j(\mathcal{O}_{S_i}(K_i)); P) \rightarrow H^n(Gl_j(K); P) \rightarrow \varprojlim_{i \in \mathbb{N}} H^n(Gl_j(\mathcal{O}_{S_i}(K_i)); P)$$

By a result of Borel and Serre ([BS]) the groups $Gl_j(\mathcal{O}_{S_i}(K_i))$ are virtual duality groups of dimension $n_i = vcd(Gl_j(\mathcal{O}_{S_i}(K_i)))$, and therefore

$$H^m(Gl_j(\mathcal{O}_{S_i}(K_i)); P) = 0 \text{ for } m \neq n_i.$$

Since $\lim_{i \rightarrow \infty} n_i = \infty$ we infer from (3.3) that $H^n(Gl_j(K); P) = 0$ for all n , proving our assertion.

Section 4 : A comparison with the Benson-Carlson groups.

Let G denote an arbitrary group and M, N be two G -modules. The group of projective homotopy classes $[M, N]$ is, by definition, the factor group of $Hom_G(M, N)$ modulo the subgroup consisting of those G -homomorphisms $M \rightarrow N$, which may be factored through a projective module. The functor Ω induces a homomorphism $[M, N] \rightarrow [\Omega M, \Omega N]$ and one can define functors $BC^n(G; ?)$, $n \in \mathbb{Z}$, by putting

$$BC^n(G; M) = \varinjlim_{k, k+n \geq 0} [\Omega^{n+k}\mathbb{Z}, \Omega^k M].$$

It was observed in [BC] that for groups G of finite virtual cohomological dimension one has $BC^n(G, M) \cong \hat{H}^n(G; M)$, which are just the Farrell cohomology groups. To deal with the case of an arbitrary group G , we first define a natural transformation $\hat{H}^n(G; ?) \rightarrow BC^n(G; ?)$. If one uses for the definition of $H^n(G; ?)$ the projective resolution

$$(4.1) \quad \dots \rightarrow P^n \rightarrow P^{n-1} \rightarrow \dots \rightarrow P^0 \rightarrow \mathbb{Z}$$

with $im(P^n \rightarrow P^{n-1}) = \Omega^n \mathbb{Z}$, $n \geq 1$, then we see that there is a natural surjective homomorphism

$$H^n(G; M) \rightarrow [\Omega^n \mathbb{Z}, M], \quad n \geq 0.$$

Passing to limits, one obtains a surjective map

$$(4.2) \quad \varinjlim_{k \geq |n|} H^{n+k}(G; \Omega^k M) \rightarrow \varinjlim_{k \geq |n|} [\Omega^{n+k}\mathbb{Z}, \Omega^k M]$$

which is well-defined for any $n \in \mathbb{Z}$. Note that the image of the connecting homomorphism $H^{n+k}(G; \Omega^k M) \rightarrow H^{n+k+1}(G; \Omega^{k+1} M)$ associated with the short exact sequence $\Omega^{k+1} M \rightarrow F\Omega^k M \rightarrow \Omega^k M$ is, by definition, equal to $S^{-1}H^{n+k+1}(G; \Omega^k M)$ and, by shifting dimensions, $S^{-1}H^{n+k+1}(G; \Omega^k M) \cong S^{-k}H^{n+k+1}(G; \Omega M)$ so that

$$(4.3) \quad \varinjlim_{k \geq |n|} H^{n+k}(G; \Omega^k M) \cong \hat{H}^{n+1}(G; \Omega M).$$

Using the natural isomorphisms $\hat{H}^n(G; M) \cong \hat{H}^{n+1}(G; \Omega M)$, we see that (4.2) together with (4.3) gives rise to a surjection, natural in M ,

$$\theta^n(G; M) : \hat{H}^n(G; M) \rightarrow BC^n(G; M),$$

which is defined for every $n \in \mathbb{Z}$.

Theorem 4.4. *The natural transformations $\theta^n(G; ?) : \hat{H}^n(G; ?) \rightarrow BC^n(G; ?)$ are equivalences for all $n \in \mathbb{Z}$.*

Proof. Only the injectivity of $\theta^n(G; M)$ needs still to be checked. Let $\bar{x} \in \hat{H}^n(G; M)$ be in the kernel of θ^n so that it may be represented by an $\bar{x} \in H^{n+k}(G; \Omega^k M)$ for some $n+k > 0$ such that the image of \bar{x} in $[\Omega^{n+k}\mathbb{Z}, \Omega^k M]$ is zero. Using the resolution (4.1) we can represent \bar{x} by a cocycle $x : P^{n+k} \rightarrow \Omega^k M$, which factors

through $\Omega^{n+k}\mathbb{Z} \subset P^{n+k-1}$, yielding a representative $y : \Omega^{n+k}\mathbb{Z} \rightarrow \Omega^k M$ of the image of \bar{x} in $[\Omega^{n+k}\mathbb{Z}, \Omega^k M]$. Consider the commutative diagram

$$\begin{array}{ccccccc}
 P^{n+k+1} & \longrightarrow & \Omega^{n+k+1}\mathbb{Z} & \longrightarrow & P^{n+k} & \longrightarrow & \Omega^{n+k}\mathbb{Z} \\
 z \downarrow & & \downarrow \Omega y & & \downarrow & & \downarrow y \\
 \Omega^{k+1}M & \xlongequal{\quad} & \Omega^{k+1}M & \longrightarrow & F\Omega^k M & \longrightarrow & \Omega^k M.
 \end{array}$$

Since by our assumption y factors through a projective module, it may be factored through $F\Omega^k M \rightarrow \Omega^k M$. But this implies that Ωy may be extended over $\Omega^{n+k+1}\mathbb{Z} \rightarrow P^{n+k}$ and thus the cocycle z representing $\delta\bar{x} \in H^{n+k+1}(G; \Omega^{k+1}M)$ is actually a coboundary. Therefore $\bar{\bar{x}} = 0$, proving the theorem.

Remark. By transport of structure, one can use the equivalences of (4.4) to define a $(-\infty, +\infty)$ -cohomological functor $BC^\bullet = \{BC^n(G; ?); n \in \mathbb{Z}\}$, equivalent to the P -completion of ordinary cohomology. The resulting connecting homomorphisms, associated with short exact sequences $A' \rightarrow A \rightarrow A''$, correspond then to maps $BC^n(G; A'') \rightarrow BC^{n+1}(G; A')$ induced from the obvious maps $[\Omega^{n+k}\mathbb{Z}, \Omega^k A''] \rightarrow [\Omega^{n+k+1}\mathbb{Z}, \Omega^k A']$, which are defined as soon as $n+k$ and k are both ≥ 0 .

REFERENCES

- [BC] D.J.Benson and J.F.Carlson, *Products in negative cohomology*, J. Pure Appl. Algebra **82** (1992), 107–129.
- [BS] A.Borel and J-P.Serre, *Cohomologie d'immeubles et de groupes S -arithmétique*, Topology **15** (1976), 211–232.
- [BG] K.S.Brown and R.Geoghegan, *An infinite-dimensional torsion-free FP_∞ group*, Invent. math. **77** (1984), 367–381.
- [CE] H.Cartan and S.Eilenberg, *Homological algebra*, Princeton University Press, Princeton, 1956.
- [F] F.T.Farrell, *An extension of Tate cohomology to a class of infinite groups*, J. Pure Appl. Algebra **10** (1977), 153–16.
- [GG] T.V.Gedrich and K.W.Gruenberg, *Complete cohomological functors on groups*, Topology and its Applications **25** (1987), 203–223.
- [HS] P.J.Hilton and U.Stammbach, *A course in homological algebra*, Graduate Texts in Mathematics, Springer Verlag, 1970.
- [I] B.M.Ikenaga, *Homological dimension and Farrell cohomology*, J. Pure Appl. Algebra **87** (1984), 422–457.
- [M] J.Milnor, *On the homology of Lie groups made discrete*, Comm. Math. Helv. **58** (1983), 73–85.

NOVEMBER 1992

E-mail address: mislin@math.ethz.ch and mislin@math.ohio-state.edu