Elementary Number Theory - Exercise 10b

ETH Zürich - Dr. Markus Schwagenscheidt - Spring Term 2023

Problem 1. Show that $\pi = [3, 7, 15, 1, \dots]$ and use it to find the rational approximation

$$\frac{355}{113} = 3.14159292035398$$

of $\pi = 3.14159265358979....$

Problem 2. Compute the continued fraction expansions of $\sqrt{5}$ and the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$.

Problem 3. Which quadratic irrational has the continued fraction expansion $[1, \overline{6, 2}]$?

Problem 4. Compute the continued fraction expansion of $\sqrt{10}$ and use it to determine the fundamental solution of Pell's equation $x^2 - 10y^2 = 1$.

Problem 5 (sage)**.** Write a program that computes the continued fraction expansion of $\sqrt{d}$ and gives a fundamental solution to Pell's equation. Verify that

$$\sqrt{61} = [7, \overline{1, 4, 3, 1, 2, 2, 1, 3, 4, 1, 14}]$$

and solve Pell's equation $x^2 - 61y^2 = 1$.