

Elementary Number Theory - Exercise 11b  
ETH Zürich - Dr. Markus Schwagenscheidt - Spring Term 2023

**Problem 1.** Show that 2023 is a congruent number.

**Problem 2.** In this problem, we show that 13, 14, 15 are congruent numbers, using different approaches.

1. Show that 13 is a congruent number, using the triangle with side lengths

$$(104329, 23400, 106921).$$

2. Show that 14 is a congruent number, using that  $(x, y) = (18, 48)$  is a rational point on the elliptic curve  $y^2 = x^3 - 14^2x$ .
3. Show that 15 is a congruent number by finding a Pythagorean triple  $(a, b, c)$  with area  $ab/2 = 60$ .

**Problem 3.** Use Tunnell's Theorem to determine the congruent numbers  $\leq 15$ .

**Problem 4.** Show that, if the converse of Tunnell's Theorem can be proved to be true (e.g. if the weak BSD conjecture is true), then every natural number  $n \equiv 5, 6, 7 \pmod{8}$  is a congruent number.

**Problem 5** (sage). Using Tunnell's Theorem, determine the congruent numbers  $\leq 100$ . For each of the numbers which might be congruent, find a suitable rational right triangle to verify that they are indeed congruent.