

Elementary Number Theory - Exercise 9a  
ETH Zürich - Dr. Markus Schwagenscheidt - Spring Term 2023

**Problem 1.** Compute the reduced representative of the form  $Q = [5, 6, 3]$ .

**Problem 2.** Compute the class number  $h(-8)$ .

**Problem 3.** Let  $p$  be an odd prime. Show that

$$p = x^2 + 2y^2 \quad \Leftrightarrow \quad p \equiv 1 \pmod{8} \quad \text{or} \quad p \equiv 3 \pmod{8}.$$

*Hint:* Rewrite the condition on the right in terms of the Legendre symbol  $\left(\frac{-2}{p}\right)$ .

**Problem 4.** Show that the class number  $h(D)$  for  $D < 0$  can become arbitrarily large.

*Hint:* Choose  $D = -4p_1 \cdots p_n$  with different primes  $p_j$ , and consider the forms  $[a, 0, c]$ .

**Problem 5.** Let  $Q = [a, b, c]$  be positive definite of discriminant  $D < 0$ . Show that, if  $a < \sqrt{-D/4}$  and  $-a < b \leq a$ , then  $Q$  is already reduced.

**Problem 6.** Show that, if  $Q$  represents 1, then  $Q$  is equivalent to the principal form.

*Hint:* Use Problem 5 from exercise sheet 8b.

**Problem 7** (sage). Write a program which, given a discriminant  $D < 0$ , computes the reduced forms of discriminant  $D$  and the class number  $h(D)$ . Use it to list the class number  $h(D)$  for  $0 > D \geq -100$ .