Elementary Number Theory

Markus Schwagenscheidt
ETH Zürich

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Elementary Number Theory - Organisation

- ▶ **Website of the seminar:**
  www.markus-schwagenscheidt.de $\rightarrow$ Teaching
- ▶ **Email:** mschwagen@ethz.ch
- ▶ **Meetings** on Wednesdays and Fridays 12:15-14:00
  (no lectures at easter; 13 weeks).
- ▶ **Script** for the lecture will be available on the website.
- ▶ **Zoom** will be used if I cannot make it to ETH.
- ▶ **Exercise sheets** will be put on the website the days before classes.
- ▶ **Solutions** will be uploaded on fridays in the next week.
- ▶ **Oral exam** of 20 minutes at the end of the semester.
  Please register for exam in mystudies during the registration period!
- ▶ **Consultation hours:** just write an email if you want to meet, or ask
  me in class.

Lectures and exercise classes

- We have two **meetings** per week, each consisting of **45 minutes lecture** and **45 minutes exercises**.
- In the **lectures** I will omit many proofs.
- The **exercises** are an **integral part** of the course.
- The exercise problems are meant to be solved **during the exercise class** and **in small groups**.
- There are usually **too many problems** on the sheet.
- There will be (optional) **homework problems**.
- There will be (optional) **sageMath problems** to get used to working with computer algebra systems.

General recommendations

▶ **Ask questions** if I'm going too fast or if I should explain something in more detail.
▶ **Work in teams** for the exercise classes, but also when recalling the lectures!
▶ **Do a lot of exercises!**
▶ **Google** questions or exercises you cannot solve; use **Wolframalpha**.
▶ **Implement** as many of the results as possible in a computer algebra system (check out **SageMath** or its online version **CoCalc**).
▶ Watch **Youtube videos** about number theory (for example, **Numberphile**'s video on the Prime Number Theorem).

Overview

- **Elementary Number Theory** mainly deals with questions about **integers**.
- "Elementary" means that the problems can be stated in elementary terms, and the proofs only use elementary methods, e.g. no complex analysis.
- However, elementary proofs are often difficult or need some ingenious tricks.
- Typical questions are:
  - What can we say about **prime numbers**?
  - Which numbers are **special**, i.e. more interesting than others? For example, when is the Mersenne number $2^n - 1$ a prime?
  - Can we **represent** each natural numbers as a sum of two squares,

  $$n = x^2 + y^2?$$

  - How to solve **polynomial equations**, e.g. $x^2 - 2y^2 = 1$ in integers? Can we describe all solutions?
  - How to solve **polynomial congruences**, e.g. $x^2 = 5 \pmod 7$?