Exercises in Convex Optimization
Lecture 1

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Open sets and algebraically open sets Let E be a real vector space. A set $S \subseteq E$ is said to be *algebraically open* if the intersection of S and any straight line of E is open according to the induced topology on the line (recall that the empty set is itself open).

1. Assume that E is a normed space. Prove that if S is open, then it is algebraically open. Prove that the converse holds when E is finite dimensional and S is convex.
2. Find an example of a (obviously non-convex) set in $\mathbb{R}^2$ that is algebraically open but not open with the usual Euclidean topology of $\mathbb{R}^2$.

`Lecture_1/AlgebraicallyOpenSets`

Convex hulls and Minkowski sums Let $A, B \subseteq \mathbb{R}^n$ be two nonempty sets (not necessarily convex). Show that $\mathrm{conv}(A + B) = \mathrm{conv}(A) + \mathrm{conv}(B)$.

`Lecture_1/ConvexHullsAndMinkowskiSums`

Liminf of convex sets Let $S_1, S_2, S_3, \ldots$ be a sequence of convex sets. Show that

$$S := \liminf_{i\to\infty} S_i := \bigcup_{k=1}^{\infty} \bigcap_{i\geq k} S_i$$

is a convex set.

`Lecture_1/LiminfConvexSets`