Exercises in Convex Optimization
Lecture 2

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Strong Separation Theorem Let us consider two *closed*, *convex* sets $A, B \subseteq \mathbb{R}^n$ with A *compact*.

(a) Show that $A - B$ is convex and closed.
Give a counterexample (with A, B disjoint, closed and convex), that compactness is needed in order to ensure that $A - B$ is closed.

(b) Suppose that A and B are disjoint. Show that those two sets can be *strongly* separated. (*Hints:* analogously to what we have done for the "open-set case" in the lecture, the proof consists again in two steps: first show that we can strongly separate $A - B$ from 0, and then use this strong separation to obtain the desired strong separation for A and B. For the strong separation of $A - B$ and 0, note that there exists an open ball centered in 0 and disjoint from $A - B$.)

`Lecture_2/ExtensionSeparationTheorem`

An alternative definition of convexity
Let $I \subseteq \mathbb{R}$ be an interval. Prove that a univariate function $f : I \to \mathbb{R}$ is convex if and only if for every $\alpha \in \mathbb{R}$, the function $t \mapsto f(t) + \alpha t$ attains its maximum over any compact subinterval $[a, b]$ of I at one of its extremities a or b.

`Lecture_2/AnotherConvexityDefinition`

Maximal Eigenvalue Let $\lambda_{\max} : \mathbb{S}^n \mapsto \mathbb{R}$ be the maximum eigenvalue function of a real symmetric matrix. Prove that $\lambda_{\max}$ is a convex function.

`Lecture_2/MaximalEigenvalue`