Problem Set 10

Due on May 18

1. Let $W, M \colon [0,1]^2 \to [0,1]$ be the lower and upper Fréchet–Hoeffding bounds. Is

$$C(u) = \lambda W(u) + (1 - \lambda)M(u)$$

   for all $\lambda \in [0,1]$ a copula? Explain your answer.

2. Let (X, Y) be a two-dimensional random vector with $\mathrm{Exp}(1)$-marginals and copula

$$C(u, v) = uv + (1 - u)(1 - v)uv.$$

   a) Does C have a density? If yes, can you compute it?

   b) Calculate the cdf of (X, Y).

   c) Does (X, Y) have a density? If yes, can you compute it?

3. Let (X, Y) be a two-dimensional random vector with cdf

$$\frac{1 - e^{-x} - e^{-y} + e^{-x-y}}{1 - e^{-x-y}}, \quad x, y \geq 0.$$

   a) Does (X, Y) have a density? If yes, can you compute it?

   b) What are the distributions of X and Y?

   c) Do X and Y have densities? If yes, can you compute them?

   d) Compute a copula C of (X, Y). Is it unique?

   e) Does C have a density? If yes, can you compute it?