Quantitative Risk Management | ETH Zurich | March 24, 2021

Problem Set 4

Due on March 31

1. Consider a random loss of the form $L = 1000(X+Y)$, where X und Y are the outcomes of two rolls of a fair dice. Calculate $\mathrm{VaR}_{0.95}(L)$, $\mathrm{ES}_{0.95}(L)$ and $\mathrm{AVaR}_{0.95}(L)$.

2. Let L be a random variable and $\alpha \in (0,1)$. Show the following:

   a)
   $$\mathrm{ES}_\alpha(L) = \mathrm{VaR}_\alpha(L) + \frac{\mathbb{E}\left[(L - \mathrm{VaR}_\alpha(L))^+\right]}{\mathbb{P}[L \geq \mathrm{VaR}_\alpha(L)]}$$

   b)
   $$\mathrm{AVaR}_\alpha(L) = \mathrm{VaR}_\alpha(L) + \frac{\int_0^1 (\mathrm{VaR}_u(L) - \mathrm{VaR}_\alpha(L))^+ \, du}{1-\alpha}$$

   c)
   $$\mathrm{AVaR}_\alpha(L) = \mathrm{VaR}_\alpha(L) + \frac{\mathbb{P}[L \geq \mathrm{VaR}_\alpha(L)]}{1-\alpha}\left(\mathrm{ES}_\alpha(L) - \mathrm{VaR}_\alpha(L)\right)$$

   d)
   $$\mathrm{AVaR}_\alpha(L) \geq \mathrm{ES}_\alpha(L) \text{ with equality if and only if } \mathbb{P}[L \geq \mathrm{VaR}_\alpha(L)] = 1-\alpha$$

   e)
   $$\mathrm{AVaR}_\alpha(a+bL) = a + b\,\mathrm{AVaR}_\alpha(L) \text{ and } \mathrm{ES}_\alpha(a+bL) = a + b\,\mathrm{ES}_\alpha(L) \text{ for all } a \in \mathbb{R} \text{ and } b > 0$$

   f)
   $$\mathrm{AVaR}_\alpha(L) \geq \mathrm{AVaR}_\beta(L) \text{ and } \mathrm{ES}_\alpha(L) \geq \mathrm{ES}_\beta(L) \text{ for } \quad \alpha \geq \beta.$$

3. Name some stylized facts of univariate financial return series.

4. Name some stylized facts of multivariate financial return series.