Problem Set 6

Due on April 13

1. Calculate the density of a standard GEV distribution H_ξ.

2. Calculate the inverse of a GEV cdf $H_{\xi,\mu,\sigma}$.

3. Show that for $x \to \infty$, one has

$$1 - H_\xi(x) \sim \begin{cases} e^{-x} & \text{if } \xi = 0 \\ (\xi x)^{-1/\xi} & \text{if } \xi > 0. \end{cases}$$

4. Show that for all $\theta \in (0,1)$, H_ξ^θ is of the same type as H_ξ.

5. Let X be a non-negative random random variable with cdf

$$F_X(x) = \frac{x}{x+1}, \quad x \geq 0.$$

   a) Does X have a density? If yes, can you derive it?

   b) Find all $k \in \mathbb{N} = \{1, 2, \dots\}$ such that $\mathbb{E}[|X|^k] < \infty$.

   c) Does F_X belong to $\mathrm{MDA}(H_\xi)$ for a standard GEV distribution H_ξ? If yes, what is ξ and what are the normalizing sequences?