Problem Set 7

Due on April 20

1. Calculate the density of a generalized Pareto distribution $G_{\xi,\beta}$.

2. Let X be a non-negative random random variable with cdf

$$F_X(x) = \frac{x}{x+1}, \quad x \geq 0.$$

   a) Calculate the excess distribution function $F_u(x) = \mathbb{P}[X - u \leq x \mid X > u]$, $x \geq 0$.

   b) Does there exist a parameter $\xi \in \mathbb{R}$ and a function β such that

$$\lim_{u \to \infty} \sup_{x > 0} |F_u(x) - G_{\xi,\beta(u)}(x)| = 0,$$

   where $G_{\xi,\beta}$ denotes the cdf of a GPD? If yes, for which ξ and β does this hold?

3. Let X be a non-negative random random variable with cdf

$$F_X(x) = 1 - x^{-4}, \quad x \geq 1.$$

   a) Does X have a density? If yes, can you derive it?

   b) Find all $k \in \mathbb{N} = \{1, 2, \dots\}$ such that $\mathbb{E}[|X|^k] < \infty$.

   c) Does F_X belong to $\text{MDA}(H_\xi)$ for a standard GEV distribution H_ξ? If yes, what is ξ and what are the normalizing sequences?

   d) Calculate the excess distribution function $F_u(x) = \mathbb{P}[X - u \leq x \mid X > u]$, $x \geq 0$.

   e) Does there exist a parameter $\xi \in \mathbb{R}$ and a function β such that

$$\lim_{u \to \infty} \sup_{x > 0} |F_u(x) - G_{\xi,\beta(u)}(x)| = 0,$$

   where $G_{\xi,\beta}$ denotes the cdf of a GPD? If yes, for which ξ and β does this hold?