Problem Set 8

Due on April 27

1. Consider a two-dimensional random vector (X, Y) with density

$$f_{X,Y}(x, y) = c\,\frac{y}{x^3} 1_{\{0<x\leq 1\}} 1_{\{0<y\leq x^2\}}.$$

   a) Calculate the constant c.

   b) Calculate the density f_X of X.

   c) Calculate $\mathbb{P}[X \leq 1/2]$.

   d) Calculate $\mathbb{E}[X/Y]$

   e) Calculate $\mathbb{E}[Y \mid X = 1/2]$

   f) Are X and Y independent?

2. Calculate

$$\int_{-\infty}^{\infty} e^{-x^2/2} dx.$$

3. Show that for a d-dimensional random vector X, one has

$$X \sim N_d(\mu, \Sigma) \quad \Leftrightarrow \quad v^T X \sim N(v^T\mu, v^T\Sigma v) \quad \text{for all } v \in \mathbb{R}^d.$$

4. Give an example of a two-dimensional random vector (X, Y) such that X and Y are normal but (X, Y) is not bivariate normal.

5. Let $X_1, \ldots, X_k$ be iid exponential random variables. Calculate the density of $X_1 + \cdots + X_k$.

6. Let $Z_1, \ldots, Z_k$ be independent standard normal random variables. Calculate the density of $Z_1^2 + \cdots + Z_k^2$.