Quantitative Risk Management | ETH Zurich | April 27, 2021

# Problem Set 9

Due on May 4

1. a) Let $Y_i \sim S_d(\psi_i)$, $i = 1, \dots, n$, be independent spherically distributed random vectors and $\alpha_1, \dots, \alpha_n \in \mathbb{R}$. Show that $Z = \sum_{i=1}^n \alpha_i Y_i$ is again spherically distributed.

   b) Assume that the daily losses of an investment during the next $t$ days are given by $(X_1, \dots, X_t) \sim M_t(0, \Sigma, \widehat{F}_W)$ for a non-negative random variable $W$ and a $t \times t$ matrix $\Sigma = \sigma^2 P$, where $\sigma > 0$ is a constant and $P$ a correlation matrix with $P_{ij} = \rho$ for all $i \neq j$. Show that there exists a function $f \colon \mathbb{N} \to \mathbb{R}$ such that

   $$\mathrm{VaR}_\alpha(X_1 + \cdots + X_t) = f(t)\mathrm{VaR}_\alpha(X_1)$$

   for all $\alpha \in (0,1)$. Can you compute $f$ explicitly?

   c) Let $X \sim E_d(\mu, \Sigma, \psi)$ and $Y \sim E_d(\nu, \Sigma, \varphi)$ be two independent elliptical random vectors. Is $Z = X + Y$ again elliptically distributed? If yes, derive $m \in \mathbb{R}^d$, $M \in \mathbb{R}^{d \times d}$ and $\xi \colon \mathbb{R}_+ \to \mathbb{R}$ such that $Z \sim E_d(m, M, \xi)$. If no, give a counterexample.

2. a) Does there exist a two-dimensional random vector with a $t_2(\nu, \mu, \Sigma)$-distribution such that $\Sigma$ is invertible and the components are independent of each other?

   b) Does there exist a two-dimensional random vector with a $t_2(\nu, \mu, \Sigma)$-distribution such that the components are independent of each other?

   c) Does there exist a two-dimensional random vector such that both components have a standard one-dimensional $t$-distribution and are independent of each other?

   d) Does there exist a $t_2(\nu, \mu, \Sigma)$-distribution that is spherical?

3. Give a two-dimensional elliptical distribution that is not a normal or a $t$-distribution.