Heinz Hopf Prize 2015

Die Eidgenössische Technische Hochschule Zürich verleiht Claire Voisin (Institut de Mathématiques de Jussieu, Paris) den Heinz-Hopf-Preis 2015 für ihre herausragenden und grundlegenden Beiträge zur komplexen algebraischen Geometrie.

L'École Polytechnique Fédérale de Zurich a l'honneur de remettre à Claire Voisin (Institut de Mathématiques de Jussieu, Paris) le prix "Heinz Hopf 2015" pour ses contributions exceptionnelles et fondamentales à la géométrie algébrique complexe.

Claire Voisin

- 1986 Ph.D., Université de Paris-Sud (adviser A. Beauville)
- 1986 Chargé de Recherche, CNRS
- 1995 Directrice de Recherche, CNRS
- Positions at École Polytechnique, IHES, and Jussieu

Previous Honors

- 1992 EMS Prize
- 2008 Clay research award
- 2010 Elected member of the French academy of sciences
- Plenary speaker at ICM 2010 (Hyderabad)

Contributions in three major directions:

- Kähler geometry: *counterexample to Kodaira's conjecture*,
- Projective geometry: *Green's conjecture for general curves*,
- Birational geometry: *the study of stable rationality*.

Voisin is an expert on algebraic cycles, Hodge theory, special geometries (abelian varieties, K3 surfaces), and the topology of algebraic varieties.

Her research yields new and fundamental results about geometries that we encounter on a daily basis: hypersurfaces, branched covers, configuration spaces, etc.

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A Kähler manifold is a complex manifold with a certain Hermitian metric (analogous to the Fubini-Study metric on $\mathbb{CP}^n$).

Are all compact Kähler manifolds deformable to algebraic varieties?

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Theorem (Voisin 2003). **No**.

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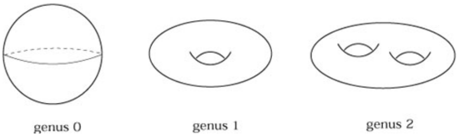
Are all compact Kähler manifolds deformable to algebraic varieties?

Theorem (Voisin 2003). **No**.

- Voisin constructs examples by blowing-up complex tori.
- Voisin's examples do not even have the homotopy type of a complex projective manifold.

§ Greens's conjecture (1983)

A curve is a complex algebraic variety of dimension 1.



Green's conjecture connects the geometry of a curve $\mathbf{C}$,

in particular the lowest degree of a cover

$$\mathbf{C} \to \mathbb{CP}^1 \ ,$$

to the algebra of the defining polynomial equations for $\mathbf{C}$.

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• Voisin (2002, 2005) proved the conjecture is true for the generic curve **C** of any genus.

• Vosin's papers (using curves on $K3$ surfaces) opened the field to substantial further progress on Green's conjecture, but the statement for every curve is still open.

§ Stable rationality

A rational variety $\mathbf{X}$ has a global algebraic coordinate system

$$\mathbb{C}^n \dashrightarrow \mathbf{X} .$$

The boundary separating rationality and irrationality is subtle:

rational $\Rightarrow$ stably rational $\Rightarrow$ unirational $\Rightarrow$ rationally connected . . .

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A new technique (via Chow decomposition of the diagonal) was introduced in

- C. Voisin, *Unirational threefolds with no universal codimension 2 cycle*, Invent. Math. (2015).

to show varieties are not stably rational.

A stably rational variety **X** has a global algebraic coordinate system after more variables are added

$$\mathbb{C}^{n+m} \dashrightarrow \mathbf{X} \times \mathbb{C}^m .$$

- Voisin's method shows several types of Fano varieties known not to be rational are in fact not stably rational.

- Explosion of activity in the past two years using Voisin's methods: Beauville, Colliot-Thélène, Hassett, Kresch, Pirutka, Schreieder, Tasin, Totaro, Tschinkel ...

If you would like to study these subjects, you can start here:

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