

Notes on a new relation in $RH^9(\bar{M}_{2,14})$

S. Canning, B. Kong, RP, D. Petersen
A. Pixton, J. Schmitt

30 September 2026 (+AI)

We follow the standard
notation for the Strata algebra

$S_{2,14}^*$ of $\bar{M}_{2,14}$

There is an Σ_{14} action on $S_{2,14}^*$.

We define

$$Q \in \left(S_{2,14}^* \right)^{[2^7]}$$

$\nwarrow$
 Σ_{14} -rep
Summand

- Color the markings

$$\{1, 2, \dots, 7\} \quad \{8, 9, \dots, 14\}$$

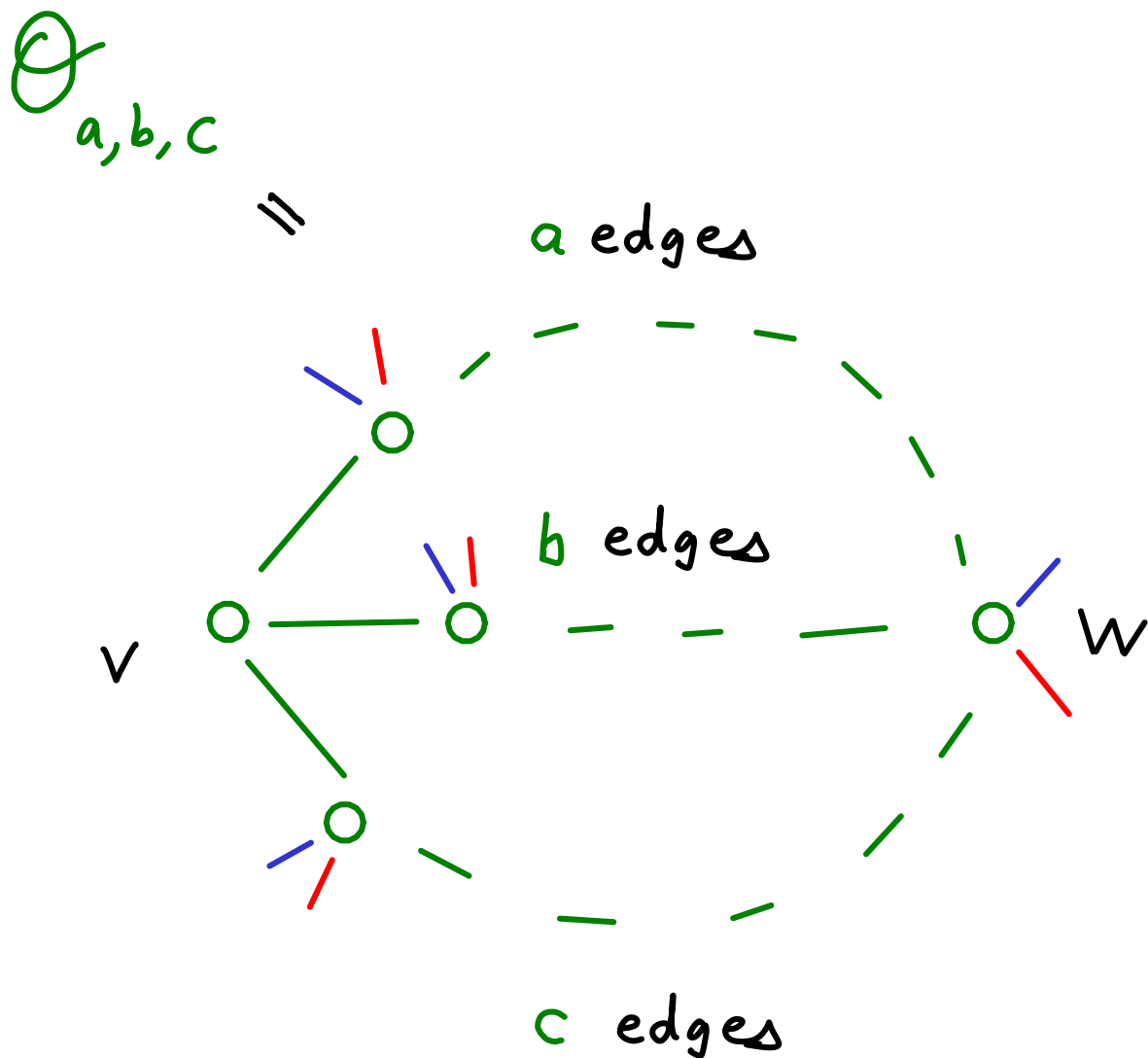
Red Blue

- For $a+b+c=9$, $a, b, c \in \mathbb{Z}_{\geq 0}$

Define a graph $\mathcal{O}_{a,b,c}$

with 8 vertices, 9 edges

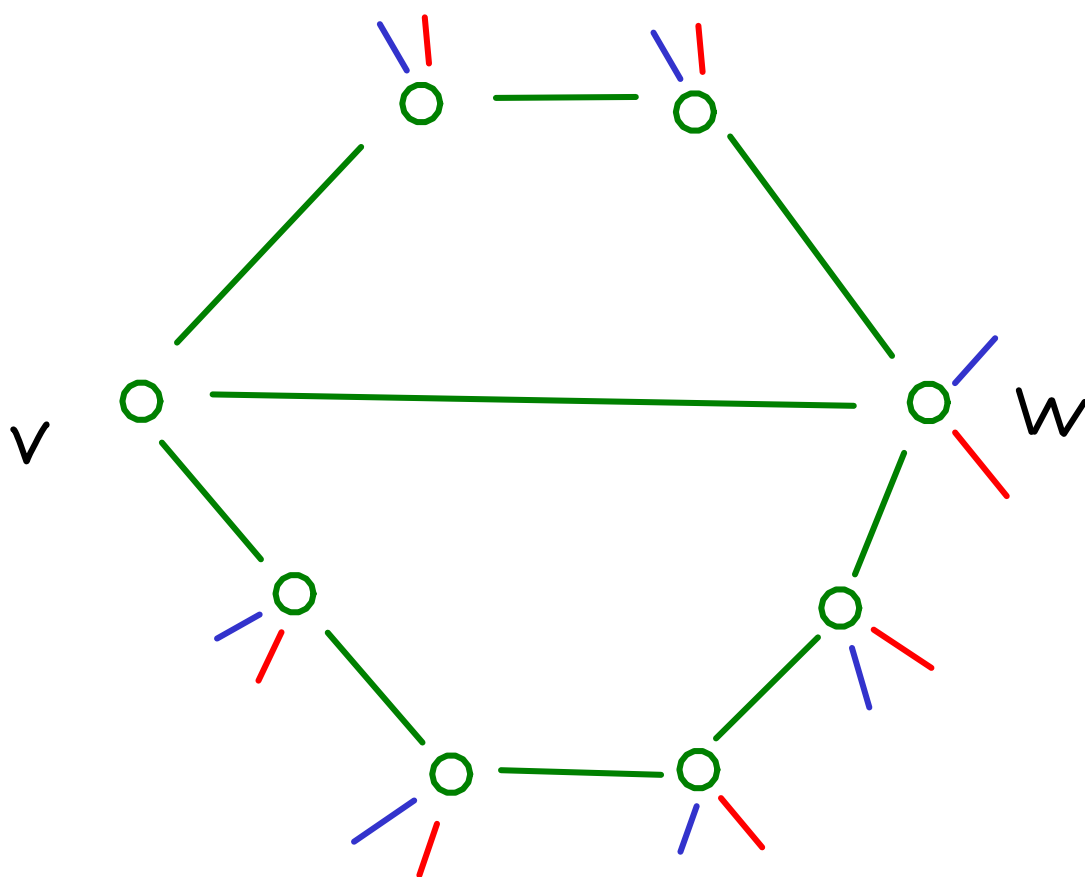
7 red legs, 7 blue legs:



- all vertices other than v carry a red leg and a blue leg.
- all vertices have genus 0.

$\mathcal{O}_{1,3,5}$

$\equiv$



- Define $[\Theta_{a,b,c}] \in S_{2,14}^9$:

Start by placing marking
 Numbers on the legs of $\Theta_{a,b,c}$
 in a way which respects the
 pairing :

$$(1, 8), (2, 9), \dots, (7, 14).$$

Then antisymmetrize with

respect to $\sum_7 \times \sum_7 \subset \sum_{14}$.

$$Q = [\Theta_{135}]$$

$$- [\Theta_{144}]$$

$$- [\Theta_{225}]$$

$$+ 2 [\Theta_{234}]$$

$$- [\Theta_{333}]$$

Project started
with AI run
by Bocho.

AI run by Johannes
used to find formula.

STEP I $Q \in \left(S_{2,14}^9 \right)^{[2^7]}$

by virtue of being the

$\Sigma_7 \times \Sigma_7$ antisymmetrization
of an Σ_2^7 invariant class.

STEP II $Q \in \left(\text{Coh Rel}_{2,14}^9 \right)^{[2^7]}$

- checked by AI software,
- Aaron found a proof by hand.

Both crucially use
that $RH^* = H^{2*}$
for $\overline{M}_{2,14}$
by Petersen-Tommasi.

The class $Q \in (S_{2,14}^9)^{[2^7]}$ yields

$$\begin{array}{ccccc}
 & \swarrow & \searrow & \searrow & \\
 Q_{\Delta_0} & \rightarrow & Q_{\Delta} & \rightarrow & 0 \\
 \uparrow & & \uparrow & & \uparrow \\
 H_{16}(\Delta_0) & \rightarrow & H_{16}(\Delta) & \rightarrow & H_{16}(\bar{M}_{2,14})
 \end{array}$$

Step III $Q_{\Delta_0} \neq 0 \in H_{16}(\Delta_0)$

- checked by AI software,
- Aaron found a proof by hand.

Step IV $Q_{\Delta} \neq 0 \in H_{16}(\Delta)$

Proof uses III and Dan's study of cohomology of local systems.

[also
checked
by AI]

[See section 5 of Dan's notes from an ETHZ lecture in 2013: [.../~rahul/DP-notes4.pdf](#).]

Step V $\left(b3\text{Spin}_{2,14}^9 \right)^{[2^7]} = \left(3\text{Spin}_{2,14}^9 \right)^{[2^7]}$

Subspace of the 3Spin relations
which arise from push-forward from
the boundary of $\bar{M}_{2,14}$.

Argument uses simple analysis
of the Σ_{14} -representation theory.

Conclusion A: $Q \notin \left(3\text{Spin}_{2,14}^9 \right)^{[2^7]}$

Because $Q \in \left(3\text{Spin}_{2,14}^9 \right)^{[2^7]}$

$\Downarrow$

$Q \in \left(b3\text{Spin}_{2,14}^9 \right)^{[2^7]}$

$\Downarrow$

$Q_\Delta = 0 \in H_{16}(\Delta).$

CONTRA
Diction!

Proof in the
end can be made
computer-free.

Conclusion B : We have

$$\begin{array}{ccccccc} (3\text{Spin}_{2,14}^g)^{[2^7]} & \subset & (\text{ChowRel}_{2,14}^g)^{[2^7]} & \subset & (\text{CohRel}_{2,14}^g)^{[2^7]} & \subset & (S_{2,14}^g)^{[2^7]} \\ \downarrow Q & & & & \downarrow Q & & \end{array}$$

• If $Q \notin (\text{ChowRel}_{2,14}^g)^{[2^7]} \Rightarrow$ cycle class map
 $(R_{2,14}^g)^{[2^7]} \rightarrow (RH_{2,14}^g)^{[2^7]}$ has a kernel.

• If $Q \in (\text{ChowRel}_{2,14}^g)^{[2^7]}$ then

$$\frac{(\text{ChowRel}_{2,14}^g)^{[2^7]}}{(3\text{Spin}_{2,14}^g)^{[2^7]}} \neq 0$$

and Pixton's conjecture fails.

Further variants and extensions of the construction will appear in the paper.