

FOCUS *on the Scientist* Tristan Rivière



Tristan Rivière, an Eisenbud Professor in this semester's *Special Geometric Structures and Analysis* program, has made outstanding contributions to geometric analysis. Recognitions of his contributions include his invited talk at the 2002 ICM in Beijing and his award of the 2003 Stampacchia medal.

Tristan spent a decade as a CNRS researcher in France and held a visiting professorship at the Courant Institute in 1999–2000. He has been a full professor at ETH Zurich since 2003, and between 2009 and 2019 has also served as FIM director, with over 50 conferences organized. He had the privilege to speak about the work of Louis Nirenberg on the occasion of Nirenberg's 2015 Abel Prize award. Tristan has now supervised over 20 Ph.D. students.

Tristan works on nonlinear PDEs and the calculus of variations, and their roles in physics and geometry. Tristan's work has elucidated many aspects of the regularity theory of nonlinear elliptic systems. To give one instance, it had long been recognized that the regularity theory for elliptic systems (as opposed to scalar equations), where the maximum principle no longer plays a decisive role, is much more delicate than for scalar elliptic PDEs. Past generations of analysts and geometric analysts had already made important contributions to this area (including such renowned figures as Charles Morrey, Ennio De Giorgi and Karen Uhlenbeck), but in many cases the optimal regularity of solutions remained unclear.

Tristan's first result on the lack of regularity for weakly harmonic maps put a stop to the search for a general regularity theory for elliptic systems with natural growth. Later on, one of Tristan's key insights has been the recognition of the power and ubiquity of conservation laws hidden within a large class of PDEs and variational problems and how such conservation laws provide a crucial new tool and unifying framework for proving optimal regularity results for such nonlinear vector-valued problems — for example, weakly harmonic maps, Willmore surfaces, and prescribed mean curvature surfaces.

Tristan's work has been deeply influenced by a variety of renowned figures in the field. Early in his career, Haïm Brezis (who served as

an unofficial Ph.D. mentor) made a strong impression, especially with his ability to distill matters to their essential ingredients and to bring the community's focus to these issues through his beautiful lectures. Another early deep and lasting influence was Karen Uhlenbeck's work on harmonic maps and Yang–Mills connections.

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Tristan's career has undergone several phase transitions. His interactions with Fang-Hua Lin opened his eyes to the realm of possibilities available when one feels free to use the whole arsenal of modern analytic techniques. Together with his work with Robert Hardt this led him to deepen his interest in regularity questions that move beyond the nonlinear PDE world to also encompass such questions in the context of geometric measure theory (GMT). Gang Tian shone a light for him on how geometry could be a source of beautiful and delicate hard analysis and GMT questions. For instance, their joint work on the regularity theory for $(1,1)$ currents in almost complex manifolds deepened his interest in analytic problems arising from calibrated geometry and its connections to gauge theory. Despite important progress in this area in recent years it remains a vast source of open problems; such problems are a central component of this semester's program.

Tristan has found analysis problems arising in geometry to be particularly fascinating, because without a sufficient apprehension of the underlying geometry it is not even possible to understand what the key analytic difficulties that one must resolve are. His current work developing the analytic aspects of min-max theory for Legendrian surfaces being the latest instance of such a geometric problem that forces one to confront new analytic challenges.

While Tristan has previously visited SLMath (MSRI as it was in 2003) for a week-long workshop, this is his first semester-long visit. He is keenly looking forward to entering into the spirit of open collaboration that SLMath encourages and thereby being exposed to new ideas, future research directions and new research collaborators.

— Mark Haskins and Costante Bellettini