ERRATA FOR
FUNCTIONAL ANALYSIS
AMS, GRADUATE STUDIES IN MATHEMATICS **191,** 2018

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Abstract. These notes correct a few typos and errors in the book *Functional Analysis*, AMS, Graduate Studies in Mathematics **191**, 2018.

p 18, l 17: Replace "$\|A(\delta\|x\|_X^{-1}x)\|_X \le 1$" by "$\|A(\delta\|x\|_X^{-1}x)\|_Y \le 1$".

p 23, l -12 to -9: The sequence x_{i_k} in the proof of Lemma 1.2.13 may not exist. To correct the argument, the sequence should be chosen as follows (thanks to Tahl Nowik for pointing out this error as well suggesting the correction).

"Choose $i_1 \in \mathbb{N}$ such that $\inf_{y\in Y}\|x_{i_1} - x_j + y\| < 2^{-1}$ for every integer $j \ge i_1$. Once $i_1, \dots, i_k$ have been constructed, choose $i_{k+1} > i_k$ to be the smallest integer bigger than i_k such that $\inf_{y\in Y}\left\|x_{i_{k+1}} - x_j + y\right\| < 2^{-k-1}$ for every integer $j \ge i_{k+1}$."

p 27, l 14: Typo: The displayed line should read

$$\|x\|_p := \left(\sum_{i=1}^{\infty}|x_i|^p\right)^{1/p} \qquad \text{for } x = (x_i)_{i\in\mathbb{N}} \in \ell^p.$$

p 111, l 11: The set $\mathcal{F} \subset \{f : X \to \mathbb{R} \,|\, f \text{ is linear}\}$ is required to be nonempty.

p 124, l 17/18: Replace $(x_{n_{i,k}})_{i\in\mathbb{N}}$, respectively $(x_{n_{i,k+1}})_{i\in\mathbb{N}}$, by $(x^*_{n_{i,k}})_{i\in\mathbb{N}}$, respectively $(x^*_{n_{i,k+1}})_{i\in\mathbb{N}}$. (Three times.)

We also remark that the Banach–Alaoglu Theorem for separable Banach spaces (Theorem 3.2.1) can be proved without using any version of the axiom of choice, as was pointed out to us by Mikhail Katz. Here is how this works.

First observe that there is a map that assigns to every bounded sequence of real numbers a convergent subsequence. Namely, given a bounded sequence $(a_n)_{n\in\mathbb{N}}$ of real numbers, let $a := \limsup_{n\to\infty} a_n$ and define a sequence $(n_i)_{i\in\mathbb{N}} =: \mathcal{T}((a_n)_{n\in\mathbb{N}})$ of positive integers recursively by $n_1 := \min\{n \in \mathbb{N} \,|\, |a_n - a| < 1\}$ and, for $i \in \mathbb{N}$,

$$n_{i+1} := \min\left\{n \in \mathbb{N} \,\middle|\, n > n_i,\, |a_n - a| < \tfrac{1}{i}\right\}.$$

Then $|a_{n_i} - a| < 1/i$ for all $i \in \mathbb{N}$ and so $\lim_{i\to\infty} a_{n_i} = a$.

Now define $(n_{i,1})_{i\in\mathbb{N}} := \mathcal{T}((\langle x_n^*, x_1\rangle)_{n\in\mathbb{N}})$ and, for $k \in \mathbb{N}$,

$$n_{i,k+1} := n_{j_i,k}, \qquad (j_i)_{i\in\mathbb{N}} := \mathcal{T}\left(\left(\langle x^*_{n_{j,k}}, x_{k+1}\rangle\right)_{j\in\mathbb{N}}\right).$$

Then the sequence $(\langle x^*_{n_{i,k}}, x_k\rangle)_{i\in\mathbb{N}}$ converges and the sequence $(x^*_{n_{i,k+1}})_{i\in\mathbb{N}}$ is a subsequence of $(x^*_{n_{i,k}})_{i\in\mathbb{N}}$ for every $k \in \mathbb{N}$.

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We also mention that the general Banach–Alaoglu Theorem (Theorem 3.2.4) is equivalent to the Boolean Prime Ideal Theorem as well as to the Tychonoff Theorem for compact Hausdorff spaces. (See the paper `http://karagila.org/wp-content/uploads/2016/10/axiom-of-choice-in-analysis.pdf` by Asaf Karagila and the references therein.)

p 367, l -4: The proof of strong continuity of the inverse semigroup uses the estimate $\sup_{0\le h\le T}\|S(h)^{-1}\| < \infty$ for all $T > 0$, which follows from the identity

$$S(h)^{-1} = S(T)^{-1}S(T-h), \qquad 0 \le h \le T,$$

the Open Mapping Theorem, and part (i) of Lemma 7.1.8.

p 388, l -2: The number $-\delta$ in equation (7.4.4) should be replaced by $+\delta$, i.e. the spectrum of A is contained in the sector

$$C_\delta := \left\{\omega_0 + re^{\mathbf{i}\theta} \,|\, r \ge 0,\ \pi/2 + \delta \le |\theta| \le \pi\right\} \tag{7.4.4}$$

(see Figure 7.4.1).

p 412, l 10: The letter "f" in equation (7.5.10) should be capitalized; thus

$$Z := \{t \in I \,|\, F \text{ is not differentiable at } t\}\,. \tag{7.5.10}$$

Here $F : I = [0,1] \to X$ is a continuous function with values in a Banach space.

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