ERRATA FOR
INTRODUCTION TO SYMPLECTIC TOPOLOGY
THIRD EDITION

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ABSTRACT. These notes correct a few typos and errors in *Introduction to Symplectic Topology* (3rd edition, Oxford University Press 2017). We thank Leo Digiosia, Katrin Wehrheim, Chris Wendl, Fabian Ziltener for pointing out errors.

p 100: The factor in equation (3.1.4) should be -1 instead of $1/2$. The correct formula is

$$\{F,\{G,H\}\}+\{G,\{H,F\}\}+\{H,\{F,G\}\}=d\tau(X_F,X_G,X_H).$$

p 109, Lemma 3.2.1: The **Moser Isotopy Lemma** can be strengthened.

Let (M,ω) be a $2n$-dimensional smooth manifold, let $Q\subset M$ be a closed submanifold, and let ω_0 and ω_1 be two symplectic forms on M that agree on T_QM. Then there exist open neighbourhoods $\mathcal{N}_0$ and $\mathcal{N}_1$ of Q and a diffeomorphism $\psi:\mathcal{N}_0\to\mathcal{N}_1$ of Q such that $\psi^*\omega_1=\omega_0$ and

$$q\in Q,\quad v\in T_qM \qquad\Longrightarrow\qquad \psi(q)=q \quad\text{and}\quad d\psi(q)v=v. \tag{1}$$

The proof does not change. The key observation is that an isotopy ψ_t satisfies (1) if and only if it is generated by a family of smooth vector fields X_t on M that satisfy

$$X_t|_Q=0,\qquad [X_t,Y]|_Q=0 \tag{2}$$

for all t and every vector field Y on M. For the 1-form σ in the proof of Lemma 3.2.1 this translates into the condition that, for every vector field Y on M, the function

$$f_Y:=\iota(Y)\sigma:M\to\mathbb{R}$$

vanishes to first order along Q, i.e.

$$q\in Q,\quad v\in T_qM \qquad\Longrightarrow\qquad f_Y(q)=0 \;\text{ and }\; df_Y(q)v=0. \tag{3}$$

The 1-form σ on a neighbourhood of Q, defined on page 110, satisfies (3) because $\partial_t\phi_t(q)=0$ and $\tau(q;v,w)=0$ for all $q\in Q$ and all $v,w\in T_qM$.

p 114, line -16: At the end of the proof of Step 3 it should be mentioned that one must use Step 1 to obtain a Hamiltonian isotopy $\{\phi_t\}_{0\le t\le 1}$ of M that satisfies $\phi_0=\mathrm{id}$ and $\phi_t\circ\Psi_0\circ\chi_{1,0}=\Psi_t\circ\chi_{1,t}$ for all t, and that this Hamiltonian isotopy satisfies the requirements of part (ii) of Theorem 3.3.1.

p 114, line -7: The term "$\Psi_t^*\omega\in\Omega^2(M)$" should read "$\Psi_t^*\omega\in\Omega^2(\mathbb{R}^{2n})$".

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p 120, Theorem 3.4.10: The proof of the **Symplectic Neighbourhood Theorem** requires the strengthened form of the **Moser Isotopy Lemma** mentioned above (see page 109).

p 129, lines 4–6: While $S(TQ^\perp)$ intersects $S(T^*L)$ in a Legendrian submanifold as claimed, a general Lagrangian submanifold of T^*L that is transverse to the unit sphere bundle $S(T^*L)$ need not intersect $S(T^*L)$ in a Legendrian submanifold.

For example, if $L = \mathbb{R}^{n+1}$ is equipped with the standard metric and $A = A^T$ is a nonzero symmetric $(n+1) \times (n+1)$-matrix, then its graph $\Lambda := \mathrm{graph}(A)$ is a Lagrangian subspace of $T^*L = \mathbb{R}^{n+1} \times \mathbb{R}^{n+1}$, transverse to the unit sphere bundle. The intersection $\Lambda \cap (\mathbb{R}^{n+1} \times S^n) = \big\{(x, y) \in \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \,|\, y = Ax,\, |Ax| = 1\big\}$ is a Legendrian submanifold of $\mathbb{R}^{n+1} \times S^n$ for the standard contact structure associated to the contact form $\alpha := \sum_i y_i dx_i$ if and only if the vectors Ax and A^2x are linearly dependent for every $x \in \mathbb{R}^{n+1}$ or, equivalently, the matrix A is a scalar multiple of an orthogonal projection. More generally, the following holds.

Let M be a contact hypersurface of a symplectic manifold (W, ω), let X be a Liouville vector field in a neighborhood of M that is transverse to M, let $\alpha := -\iota(X)\omega$ be the associated contact form, let $Y \in \mathcal{X}(M)$ be the Reeb vector field associated to α, and let $\Lambda \subset W$ be a Lagrangian submanifold that is transverse to M. Then the intersection $\Lambda \cap M$ is a Legendrian submanifold for the contact structure $\xi := \ker \alpha$ if and only if $X(q) \in T_q\Lambda + \mathbb{R}Y(q)$ for every $q \in \Lambda \cap M$.

p 147, line -19: The sentence should read *"Examples by Eliashberg* [184] *show that weak and strong fillability differ in dimension* 3 *and Massot–Niederkrüger–Wendl* [440] *proved that they differ in dimension* 5*. The question of weak versus strong fillability is open in dimensions* 7 *and higher."*

p 147, line -17: The sentence should be expanded as follows:
"By a result of Eliashberg [178] *and Gromov* [287] *overtwisted contact 3-manifolds are never weakly fillable. A similar result holds in higher dimensions by results of Niederkrüger* [N]*, Massot–Niederkrüger–Wendl* [440]*, and Borman–Eliashberg–Murphy* [75]*. The heart of the proof is a result by Niederkrüger* [N] *which asserts that a contact manifold containing a 'plastikstufe' is not strongly fillable. In Massot–Niederkrüger–Wendl* [440] *it is explained how the same argument shows that a 'small plastikstufe' obstructs weak fillability, and the existence of a 'small plastikstufe' is an easy consequence of Borman–Eliashberg–Murphy flexibility."*

p 147, last paragraph: There are some inaccuracies in the discussion of the literature. The paragraph should be rewritten as follows.
"An elementary 2-dimensional argument shows that a Liouville domain can have a disconnected (convex) boundary (Example 3.5.29 and Definition 3.5.32). That this phenomenon also occurs in higher dimensions was shown by McDuff [451] and Mitsumatsu [M] *in dimension four and by Geiges [260] in dimensions four and six. Thus fillable contact manifolds do not have to be connected. Examples in all dimensions appear in the work of Massot–Niederkrueger–Wendl [440], where they are an essential ingredient in their construction of nonfillable tight contact manifolds. Using fillable disconnected contact* 3*-manifolds, Albers–Bramham–Wendl [19] constructed examples (attributed to Etnyre) of nonseparating contact hypersurfaces in certain closed 4-dimensional symplectic manifolds. However not all contact manifolds support such an embedding, and also there are restrictions on the ambient symplectic manifold."*

p 270, lines 5-7: The factor in lines 5 and 6 should be $\lambda := 1/|z(t)|^2$. Moreover, the displayed equation in line 7 for the monodromy is incorrect. The correct formula has the form

$$z(t) = x(t) + iy(t) = e^{\pi i t}\big(u(t) + iv(t)\big), \tag{4}$$

where the function $w = u + iv : \mathbb{R} \to \mathbb{C}^n$ is the solution of the differential equation $\dot{w} = i\pi(-w + \lambda\overline{w})$ with the initial condition $w(0) = z(0) =: z =: x + iy$. It follows that the function $t \mapsto |z(t)|$ is constant and that $w(t)$ is given by

$$\begin{aligned} &\sqrt{1 - 1/|z|^2}u(t) + i\sqrt{1 + 1/|z|^2}v(t) \\ &= \exp\Big(-\pi i\sqrt{1 - 1/|z|^4}\Big)\Big(\sqrt{1 - 1/|z|^2}x(0) + i\sqrt{1 + 1/|z|^2}y(0)\Big) \end{aligned} \tag{5}$$

or, equivalently, by

$$|y|u(t) + i|x|v(t) = \exp\left(-\frac{2\pi i|x||y|}{|x|^2 + |y|^2}\right)\Big(|y|x + i|x|y\Big). \tag{6}$$

for every $t \in \mathbb{R}$. For $t = 1$ the formulas (4) and (5) together agree with the displayed equation in line 7 on page 270. Hence equation (6.3.11) is correct as stated.

p 275, lines 9 and 11: The first displayed formula should read

$$dH_b = \iota([v_1^\sharp, v_2^\sharp]^{\mathrm{Vert}})\sigma_b.$$

Moreover, in view of Lemma 6.4.8, there should be a minus sign in the second displayed formula, i.e. it should read $\tau_\Gamma(v_1^\sharp(x), v_2^\sharp(x)) := -H_{\pi(x)}(x)$.

p 280, line -13: There should be a minus sign in equation (6.4.2), i.e.

$$\tau_\Gamma(v_1^\sharp, v_2^\sharp) = -H_{v_1,v_2}. \tag{6.4.2}$$

p 281, line -10: Lemma 6.4.8 actually asserts that

$$\iota([v_1^\sharp, v_2^\sharp]^{\mathrm{Vert}}) \overset{\mathrm{fibre}}{=} -d\big(\tau(v_1^\sharp, v_2^\sharp)\big). \tag{6.4.5}$$

p 281, line -5: The displayed formula should read $\tau(v_1^\sharp, v_2^\sharp) = -H_{v_1,v_2}$ as in the corrected version of equation (6.4.2). In other words, the curvature of a closed connection 2-form τ assigns to each pair of tangent vectors $v_1, v_2 \in T_bB$ of the base the Hamiltonian vector field $[v_1^\sharp, v_2^\sharp]^{\mathrm{Vert}}$ on the fibre (F_b, σ_b) that is generated by the Hamiltonian function $H_{v_1,v_2} := -\tau(v_1^\sharp, v_2^\sharp) : F_b \to \mathbb{R}$.

p 398, line -4: Replace $\mathcal{U}$ by $\mathcal{U}_\delta$ (twice).

p 405: In Exercise 10.2.23 part (v) replace Γ_σ by Γ_{vol}.

p 425: The path $\beta : [0, 1] \to [0, 1]$ in Exercise 11.1.11 is required to satisfy the condition $\beta(0) = 0$.

p 531: The first sentence in part (ii) of Remark 13.3.28 should read: *"In* [431], *Liu also proved that a minimal closed symplectic four-manifold* (M, ω) *is rational or ruled if and only if the symplectic form* ω *is homotopic to a symplectic form* ω' *that satisfies* $K \cdot [\omega'] < 0$*."* (Ruled surfaces over curves of genus at least two admit symplectic forms ω that satisfy $K \cdot [\omega] \geq 0$.)

References

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