Errata for
Measure and Integration
EMS, Textbooks in Mathematics, 2016

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Abstract

These notes correct typos and errors in the book *Measure and Integration*, EMS, Textbooks in Mathematics, 2016. Thanks to Theo Bühler, Walter Ebinger, Patrick Nüesch, Joa Weber for pointing out errors.

p 26, l -6: Delete the word "how".

p 36: The comma at the end of line 2 should be removed and line 3 should read "by part (iii) of Theorem 1.44, it follows that".

p 39, l 2: Replace "function" by "functions".

p 53, l 7: Delete the repeated "that".

p 71, l -13: Add the words "$A_i \subset U_i$ and" after "such that".

p 71, l -8: Replace "=" by "$\leq$" in the displayed inequality. (The sequence of open sets U_i may not be ascending.)

p 74: Replace the number "2^{-k}" at the end of line -10 by "2^{-nk}"

p 90: Exercise 2.27 should read.
"Find examples of Lebesgue null sets $A, B \subset \mathbb{R}^n$ whose sum

$$A + B := \{x + y \,|\, x \in A,\, y \in B\}$$

is not a Lebesgue null set."

p 108: Two lines before the proof of Theorem 3.12: Delete one of the repeated words "the".

p 137, l -2 and l -8: Replace "$\sum_{p=1}^{\infty}$" by "$\sum_{n=1}^{\infty}$" (twice).

p 140, l -5: Insert the article "a" before "Cauchy sequence".

p 144, l -5: Replace "$\mathbb{N}$" by "$\mathbb{N}_0$".

p 144-145: The notation was modified by the copy-editor and the same letter $\mathcal{V}$ was used for the countable basis of the topology and for the associated subset $\mathscr{V} \subset \mathcal{L}^p(\mu)$ of measurable step functions with rational values. To avoid confusion the letter $\mathcal{V}$ should be replaced by $\mathscr{Q}$ twice on page 144 (lines -2, -1) and four times on page 145 (lines 1, 3, 17).

Moreover, the reader is advised that the letter $\mathcal{J}$ on page 145 (in the original text $\mathcal{I}$ was used) has a different meaning from the letter $\mathcal{J}$ on page 144 (in the original text $\mathscr{I}$ was used).

p 145, l -7: Replace "μ_1" by "μ".

p 147, l 10: Replace "if X is any set" by "if X is any infinite set".

p 161, Theorem 4.39: The second equation in (4.39) is wrong, in general, and should be replaced by the inequality

$$\max\left\{\left\|\Lambda^+\right\|, \left\|\Lambda^-\right\|\right\} \le \|\Lambda\|.$$

Thus (4.39) should read

$$\Lambda = \Lambda^+ - \Lambda^-, \qquad \max\left\{\left\|\Lambda^+\right\|, \left\|\Lambda^-\right\|\right\} \le \|\Lambda\|. \tag{4.39}$$

This weaker assertion of Theorem 4.39 suffices for the application in the proof of Theorem 4.35.

p 168: In the hint for Exercise 4.41 replace "First prove" by "Prove".

p 169: In the last line (Exercise 4.46 (c)) replace the last "q" by "p".

p 179, Definition 5.1: Replace "$\lambda : \mathcal{A} \to [0,\infty)$" by "$\lambda : \mathcal{A} \to [0,\infty]$" .

p 197: In part (i) of Lemma 5.11 replace "$\mu(\emptyset) = 0$" by "$\lambda(\emptyset) = 0$".

p 222, l 15: Replace "$\kappa(s,t)$" by "$\kappa(s,f)$"

p 229, l -3: Replace "$M\mu : \mathbb{R}^n \to \mathbb{R}$" by "$M\mu : \mathbb{R}^n \to [0,\infty]$"

p 230, l 3: Replace "$M\lambda := M|\lambda| : \mathbb{R}^n \to \mathbb{R}$" by "$M\lambda := M|\lambda| : \mathbb{R}^n \to [0,\infty]$".

p 230, l 6: Replace "$M\lambda : \mathbb{R}^n \to \mathbb{R}$" by "$M\lambda : \mathbb{R}^n \to [0,\infty]$".

p 230, l 10: Replace "and so $M\lambda \in \mathcal{L}^{1,\infty}(\mathbb{R}^n)$" by "and so $M\lambda$ agrees almost everywhere with a function in $\mathcal{L}^{1,\infty}(\mathbb{R}^n)$".

p 230, l 14: Replace "$M\mu : \mathbb{R}^n \to \mathbb{R}$" by "$M\mu : \mathbb{R}^n \to [0,\infty]$".

p 230, l 15: Replace "Let $t > 0$" by "Fix a real number $t > 0$".

p 238, l -12: Replace "$A \subset U_\delta$" by "$\mathbb{R}^n \setminus A \subset U_\delta$".

p 238, l -2: Remove the summation sign "$\sum_{i=1}^N$"

p 240, l 15: Replace "ever" by "every".

p 242, l 13-17: Exercise 2.27 was wrong and cannot be used to deduce that $f_s(E_s)$ is a Lebesgue null set. Instead, one can argue as follows. "Let $\lambda = \lambda^+ - \lambda^-$ be the Jordan decomposition in Definition 5.13. Then, by Lemma 5.16, λ^+ and λ^- are absolutely continuous with respect to the Lebesgue measure. Now define the monotone functions $f^\pm : I \to \mathbb{R}$ by

$$f^+(x) := f(a) + \lambda^+([a,x]), \qquad f^-(x) := \lambda^-([a,x])$$

for $x \in I$. Then $f = f^+ - f^-$. Moreover, by Lemma 5.21, the functions $f^\pm$ are absolutely continuous and so is the function $f_s = f^+ - f^- - f_a$. Since (i) implies (ii) (already proved), this shows that $f_s(E_s)$ is a Lebesgue null set."

p 243, l 1: Replace "$f(U_i)$" by "$f_s(U_i)$".

p 243, l 3 and l 8: Replace "$f(A_n)$" by "$f_s(A_n)$".

p 244, l 13: Replace "as in (iii)" by "as in (iv)".

p 259, l 5: The displayed formula should read

$$\int_Y \mu(\Delta^y)\, d\nu(y) = 0 \neq 1 = \int_X \mu(\Delta_x)\, d\mu(x).$$

p 259, l -10: The displayed formula should read

$$\int_Y \mu(Q^y)\, d\nu(y) = 0 \neq 1 = \int_X \mu(Q_x)\, d\mu(x).$$

p 261, l -14: Replace "nonegative" by "nonnegative".

p 265, l -3: Replace "$\mathcal{A} \times \mathcal{B}$-measurable" by "$\mathcal{A} \otimes \mathcal{B}$-measurable".

p 266, l 2: Replace "$\Phi^\pm : Y \to [0,\infty]$" by "$\Phi^\pm : X \to [0,\infty]$".

p 276, l -3: In part (iii) replace the sentence
"Let $\phi \in \mathcal{L}^1(\mu)$ and $\psi \in \mathcal{L}^1(\nu)$ be as in (i) and (ii)."
by "Let $\phi \in \mathcal{L}^1(\mathbb{R}^k)$ and $\psi \in \mathcal{L}^1(\mathbb{R}^\ell)$ be as in (i) and (ii)."

p 280, l -10: Replace "$\mathbb{R}^{3n} \to \mathbb{R} : (\xi, \eta, \zeta) \mapsto (f(\xi), g(\eta), h(\zeta))$"
by "$\mathbb{R}^{3n} \to \mathbb{R} : (\xi, \eta, \zeta) \mapsto f(\xi)g(\eta)h(\zeta)$".

p 283: Replace the last two lines by the following text.
Define $K := \{x + \xi \,|\, x, \xi \in \mathbb{R}^n \,|\, x \in \mathrm{supp}(g),\ |\xi| \le 1\}$. Since g is uniformly continuous, there exists a constant $0 < \delta \le 1$ such that, for all $\xi \in \mathbb{R}^n$,

$$|\xi| < \delta \qquad \Longrightarrow \qquad \sup_{x \in \mathbb{R}^n} |g(x+\xi) - g(x)| < \left(\frac{\varepsilon}{3^p m(K)}\right)^{1/p}$$

p 284, l 4: Replace "$m(\mathrm{supp}(g))$" by "$m(K)$".

p 285, l 8: Replace "$g(y + he_i - y)$" by "$g(y + he_i)$"

p 285, l 12: Swap "$dm(y)$" with the vertical bar in the penultimate line of the display.

p 290, l 1: Replace "$f \in \mathcal{L}^1(\mu) \cap \mathcal{L}^2(\mu)$" by "$f \in \mathcal{L}^1(\mu) \cap \mathcal{L}^q(\mu)$"

p 311, l -9: Replace "$\mathbb{N}$" by "$\mathbb{N}_0$".

p 313, l 1: Insert "of $\mathbb{1}$" after "$U_0 \subset \mathrm{G}$".

p 319, l 1: Replace "$f \in C_c^+(\mathrm{G})$" by "$f, g \in C_c^+(\mathrm{G})$".

p 327, l -13: Replace "Step 3" by "Step 4".

p 339, l 8: Replace "(F)" by "**(P)**" in the beginning of the line.