Erratum for the paper "Transversality in elliptic Morse theory for the symplectic action" by Andreas Floer, Helmut Hofer, Dietmar Salamon *Duke Mathematical Journal* **80** (1996), 251–292.

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In the proof of Lemma 6.5 on page 277, line 9, the equation should read

$$\begin{aligned}&(SJ_0 - J_0S - a - bJ_0)\xi - (\widehat{\alpha} + \widehat{\beta}J_0)\zeta \\ &= \eta_2 - (\widehat{S}_1J_0 - J_0\widehat{S}_1)\zeta - (SJ_0 - J_0S - a - bJ_0)\widehat{S}_1\zeta.\end{aligned} \tag{1}$$

(See also `https://people.math.ethz.ch/~salamond/PREPRINTS/trans.pdf`, page 25.) Here $S = S^T \in \mathbb{R}^{2n\times 2n}$, $a, b \in \mathbb{R}$, and $\zeta \in \mathbb{R}^{2n} \setminus \{0\}$ satisfy

$$(SJ_0 - J_0S - a - bJ_0)\zeta = 0, \tag{2}$$

the linear map

$$\mathbb{R} \times \mathbb{R} \times \mathbb{R}^{2n} \to \mathbb{R}^{2n} : (\widehat{\alpha}, \widehat{\beta}, \xi) \mapsto (SJ_0 - J_0S - a - bJ_0)\xi - (\widehat{\alpha} + \widehat{\beta}J_0)\zeta \tag{3}$$

is surjective, and $\widehat{S}_1 = \widehat{S}_1^T \in \mathbb{R}^{2n\times 2n}$ was chosen such that $(\widehat{S}_1J_0 - J_0\widehat{S}_1)\zeta = \eta_1$. By surjectivity of the map (3) there exists a solution $(\widehat{\alpha}, \widehat{\beta}, \xi) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{2n}$ of (1), and it follows from (2) that, if the triple $(\widehat{\alpha}, \widehat{\beta}, \xi)$ satisfies (1), then so does the triple $(\widehat{\alpha} - 2tb, \widehat{\beta} + 2ta, \xi - tJ_0\zeta)$ for every $t \in \mathbb{R}$. Hence the solution $(\widehat{\alpha}, \widehat{\beta}, \xi)$ of (1) can be chosen such that

$$\langle \xi, J_0\zeta \rangle = 0. \tag{4}$$

This implies that there exists a matrix $A \in \mathbb{R}^{2n\times 2n}$ such that

$$A\zeta = \xi, \qquad A^T = A, \qquad AJ_0 = J_0A. \tag{5}$$

(See line 11 on page 277; if (4) does not hold, then the matrix A on page 277, line 13/14, is not symmetric.) By (1) and (5) the matrix $\widehat{S} := \widehat{S}_1 + A = \widehat{S}^T$ satisfies the equations $(\widehat{S}J_0 - J_0\widehat{S})\zeta = \eta_1$ and

$$(\widehat{S}J_0 - J_0\widehat{S})\zeta + (SJ_0 - J_0S - a - bJ_0)\widehat{S}\zeta - (\widehat{\alpha} + \widehat{\beta}J_0)\zeta = \eta_2 \tag{6}$$

as claimed.