Title. On Serre's modularity conjecture.

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Let $\overline{\rho} : G_Q \to GL_2(F)$ be a continuous representation of the Galois group of Q in a 2 dimensional vector space over a finite field F of characteristic p. One supposes that $\overline{\rho}$ is absolutely irreducible and odd, *i.e.* $\overline{\rho}(c) = -id$ where c is the complex conjugation. Serre's conjecture says that $\overline{\rho}$ arises from a suitable modular form f, more precisely is the reduction modulo p of the p-adic Galois representation associated by Deligne to f.

Granted to the efforts of many mathematicians the conjecture should now be a theorem. The aim of the course is to provide an introduction to the the meaning of the conjecture and the tools for the proof :
- weak and strong form of the conjecture ; the conjecture of Fontaine and Mazur ;
- ramification and first cases, deformation of Galois representations, Lifting Modularity Theorems (Wiles, Taylor-Wiles) ;
- the strategy : the case of level 1 (Khare) and general case.

Prerequisites : basic knowledge in Number Theory and Algebraic Geometry ("Algebraic Number Theory" Neukirch, "Algebraic Geometry" Hartshorne)

Bibliography

Henri Darmon, Fred Diamond, Richard Taylor : Fermat's last theorem. Elliptic curves, modular forms and Fermat's last theorem (Hong Kong, 1993), 2–140, Int. Press, Cambridge, MA, 1997.

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Chandrashekhar Khare : Serre's modularity conjecture: the level one case. Duke Math. J. 134 (2006), no. 3, 557–589.

Kenneth Ribet, William Stein, Lectures on Serre's conjectures. Arithmetic algebraic geometry (Park City, UT, 1999), 143–232.

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