Teaching Mathematics to Engineering Students at ETH: Coping with the Diversity of Engineering Studies

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Contents of the first year (semesters 1 and 2)

The same for EE, ME, CE:

- Basic calculus in 1 dimension (differentiation, integration) with some proofs
- Basic linear algebra with proofs and algorithms

Learning goals:

- Know the concepts
- Get familiar with the mathematical objects
- Get familiar with important mathematical models used in the own engineering branch
- Learn to manipulate the relevant mathematical objects with paper and pencil and with mathematical software

Contents of the first year (Continued)

Main difficulty in calculus: Transition from 1 dimension to 2 dimensions

If this is understood, the transition to 3 and more dimensions is easy!

Example: Increment, total differential

$$\begin{array}{llll}
1\ D: & f(x), & df|_{x^0} & = f'(x^0)\,dx, \quad f'(x) = \dfrac{df}{dx} \\
2\ D: & f(x,y), & df|_{x^0,y^0} & = \dfrac{\partial f}{\partial x}(x^0,y^0)\,dx + \dfrac{\partial f}{\partial y}(x^0,y^0)\,dy \\
n\ D: & f(\mathbf{x}), & df|_{\mathbf{x}^0} & = \sum_{k=1}^{n} \dfrac{\partial f}{dx_k}(\mathbf{x}^0)\,dx_k, \quad \mathbf{x} = (x_1,\ldots,x_n)^T \\
 & & & = \langle \mathrm{grad}\, f(\mathbf{x}^0),\ d\mathbf{x} \rangle \qquad \text{(dot product)}
\end{array}$$

Mathematics in the the first two years, hours per week

Dept	Sem	Calculus	LinAlg	DiscrM	NumerAn	ComplAn	Stat
ElecE	1	4+2 / 5+3	2+1				
	2	4+2 / 5+3				4	
	3	2+1		2+1			
	4				2+1		2+1
MechE	1	8+1	2+2				
	2	8					
	3	2+1			2		2+1
CivilE	1	6	4				
	2	6					4
	3	2				4, Geom	

Mathematics in the first two years, explanations

- n + m means: n hours in class + m hours present in exercises or colloquia
- A / B means: a fast course with characteristics A and a slow course with characteristics B is offered; the student chooses
- 4 hours Geometry only for Survey Engineers (Geomatik)

Mathematics in the first two years, for comparison

Computer Science (Informatics Engineers, InfoE)

Chemical and Environmental Engineers, Pharmacists (ChemE)

Dept	Sem	Calculus	LinAlg	DiscrM	NumerAn	ComplAn	Stat
InfoE	1	4+2 / 5+3	3+1				
	2	4+2 / 5+3	3+1				3+1
	3			3+1			
	4				3+1		
ChemE	1	3+2					
	2	2+1	2+1				
	3						
	4						

Mathematics in the second year

The main topics: Differential equations

The principal mathematical models in engineering sciences!

- Ordinary differential equations (ODEs) for modelling point mechanics, electrical networks, etc.
- Partial differential equations (PDEs) for modelling electrical fields, heat conduction, wave propagation, elastic deformation, fluids, etc., etc.

The topics are selected according to the needs of the engineering departments, in collaboration between these departments and the teacher, generally a member of the mathematics department.

Care is taken to choose the examples used in the course and the exercises from the particular engineering background.

Specialized topics, varying

For everybody: Vector analysis, e.g. Gradient, potential, directional derivative, contour integral, surface integral, flow, divergence, curl.

For Electrical Engineers: Fourier series, Fourier integrals, discrete Fourier transformation (DFT), fast Fourier transformation (FFT), Laplace transformation, complex analysis, conformal mappings, potential theory, wave propagation, ODEs and PDEs.

For Mechanical Engineers: Geometry in 3 dimensions (robotics), ODEs, point mechanics, rigid-body motion, elasticity, PDEs.

For Civil Engineers: Geometry in 3 dimensions, elliptic PDEs, in particular the plate equation.

Goals and suggestions

1. Stress linear algebra, if necessary at the cost of analysis!

- The operations of calculus are defined as limits of algebraic operations acting on discrete data
- This leads to the extremely useful and elegant concepts of mathematical analysis
- Unfortunately, most problems of analysis do not have explicit solutions, i.e. solutions in terms of known functions
- Therefore, engineers – in need of concrete solutions – are forced to use discrete approximations: e.g. sums instead of integrals, systems of algebraic equations instead of PDEs, etc.
- A shortcut, partially circumventing calculus: directly solve the discrete problem by methods of linear algebra. Example: finite elements

A simpler example: Indefinite integrals

Plot the graph of

$$F(x) := \int_0^x f(t)\ dt\,, \quad f(t) = e^{-t/3}\ t\ \cos(t)$$

in the interval $0 \leq x \leq 20$.

Closed form (integration by parts or by MAPLE, MATHEMATICA, etc.):

$$F(x) = e^{-x/3}\,[(0.9\,x + 0.54)\sin(x) + (-0.3\,x + 0.72)\cos(x)] - 0.72$$

Conventional approach: Closed form by hand (tiring!), checking and plotting by mathematical software.

Direct approach: With a sufficiently small step h the subsequent trapezoidal sums

$$0, \quad \frac{h}{2} f_0 + \frac{h}{2} f_1, \quad \frac{h}{2} f_0 + h f_1 + \frac{h}{2} f_2, \quad \ldots\,, \quad f_k = f(k\,h)$$

are good enough for plotting accuracy. MATLAB code:

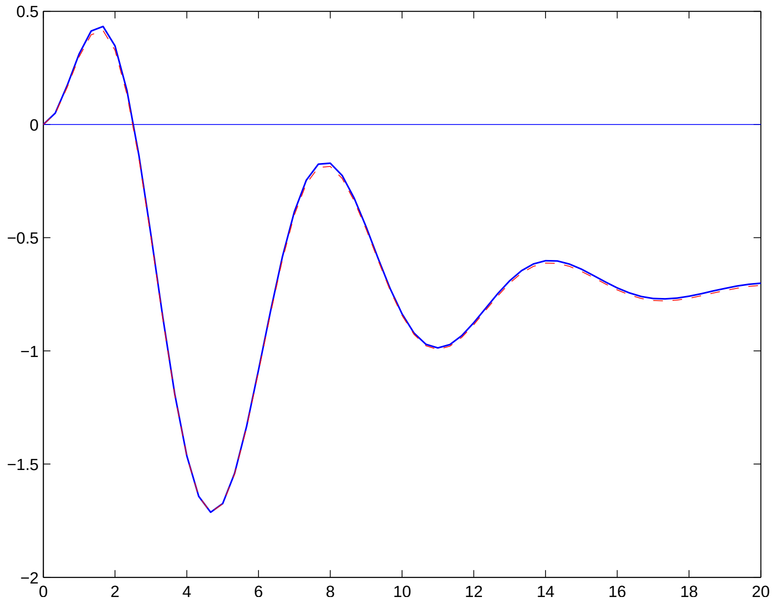
```
xf=20; h=1/3; x=0:h:xf; e=exp(-x/3); c=cos(x); f=e.*x.*c;
Fexact=e.*((0.9*x+0.54).*sin(x) + (-0.3*x+0.72).*c)-0.72;
F=h*(cumsum(f)-f/2-f(1)/2);  plot(x, Fexact, x, F, '--');
```

Numerical approach for high accuracy: Use an ODE integrator to solve the initial value problem

$$\frac{dF}{dx} = e^{-x/3}\; x\;\cos(x)\,, \quad F(0) = 0$$

Indefinite integral of exp(-x/3) x cos(x)

Solid line: exact integral, dashed: trapezoidal approximation, $h = 1/3$



($h = 1/16$ yields 4 digits of accuracy)

Goals and suggestions, continued

2. Teach numerical analysis along with calculus!

- Numerical analysis is essential for every engineer
- The practical engineering problems are rarely solvable in closed form
- Combining calculus and numerical analysis saves time!
- The capability of efficiently solving difficult math problems (and visualizing the results) increases the motivation of the engineering students
- A problem: to find teachers willing to teach such courses. Possible solution: collaboration between a mathematician and a numerical analyst

Goals and suggestions, continued

3. Appropriate use of modern mathematical software

- Mathematical software cannot replace basic understanding
- Mathematical software, in particular symbolic computation, is very useful in many situations, e.g. for checking the correctness, for shortening lengthy calculations, etc.
- Closed-form solutions – if they exist – are often too long to be useful
- Canned solutions lack transparency. The user is often reduced to a typist: Key in the problem in the syntax of the software system and type solve
- The numerical approach (e.g. with the help of MATLAB) is more transparent and more versatile.

Goals and suggestions, continued

4. Improve connection with specific engineering topics

- At ETH the topics of the math courses of the second year are chosen in agreement with the engineering departments
- Students always love examples. Primarily use examples from particular engineering fields. This requires intensive contact between the math teachers and the engineering departments
- Danger: the mathematician's ideas about an engineering example: "Consider a practical example: Let f be a Hölder-continuous function defined on $\mathbb{R}_+ \ldots$"

Conclusions

- ETH offers a diversified Mathematics Program adapted to the engineering departments
- 12 to 6 hours per week, in decreasing order: EE, ME, CE, 1st year, 2nd year
- Adapted contents mainly in the second year
- Possible improvements: Stress linear algebra, possibly include also some discrete mathematics. Mainly use examples from engineering
- Linear algebra is largely taught in combination with numerical algorithms. A great effort should be undertaken in order to extend this principle to all courses in engineering mathematics.