New trends in geometry and mathematical physics
Monte Verita, 18.08.-23.08.2019

Scientific program

Denis Bernard

How to quantise the macroscopic fluctuation theory?

The macroscopic fluctuation theory is an effective theory adapted to describe transport and its fluctuations in diffusive classical systems. We shall discuss a possible route towards the construction of a quantum analogue of the macroscopic fluctuation theory which would potentially be adapted to deal with out-of-equilibrium coherent phenomena and their fluctuations in diffusive many-body quantum systems. We shall particularly discuss a stochastic version of fermion hoping models, which is a quantum analogue of the symmetric simply exclusion process, and its exact solution.

Yuri Berest

Representation homology of spaces

Representation homology is an algebraic homology theory associated with derived representation schemes which are natural ('derived') extensions of classical representation varieties. The subject may be plainly viewed as part of derived algebraic geometry; however, somewhat surprisingly, there are more classical and elementary approaches.
In this talk, we will discuss representation homology of topological spaces, which is a homological extension of representation varieties of fundamental groups. We will give an elementary construction of this homology theory parallel to the usual (simplicial) homology of spaces. This allows us to compute representation homology explicitly (in terms of known invariants) in a number interesting cases, including spheres, suspensions, complex projective spaces, Riemann surfaces and some 3-dimensional manifolds, such as link complements and lens spaces. Time permitting, we will also discuss some conjectures relating representation homology to topology and representation theory, including (generalizaions of) the celebrated strong Macdonald conjecture and the derived Harish-Chandra conjecture.
The talk is based on joint work with A. Ramadoss and W.-K. Yeung as well as earlier works with G. Felder and T. Willwacher.

Damien Calaque

Shifted symplectic reduction of derived critical loci

In this talk we will introduce the notions of shifted symplectic groupoid and shifted symplectic reduction. We will provide examples, focusing in particular on describing equivariant derived critical loci by means of shifted symplectic reduction. If time permits, we will also explain how one can construct shifted symplectic groupoids from the AKSZ-PTVV construction. This is based on joint works with Mathieu Anel (equivariant derived critical loci) and Pavel Safronov (shifted symplectic groupoids).

Alberto Cattaneo

The Poisson sigma model and its applications

In this talk I will give a basic overview of the Poisson sigma model from Kontsevich's star product to recent developments. This covers several years of collaborations with Giovanni.

Ivan Cherednik

Motivic approach to plane curve singularities and Landau-Ginzburg models

We will begin with a mini-review of the classical theories of zeta-functions and then switch to the flagged (motivic) zeta functions of plane curve singularities and the corresponding L-functions (their numerators). Through DAHA theory, the latter conjecturally coincide with the stable Khovanov-Rozansky polynomials of the corresponding (algebraic) knots and with physics superpolynomials (from M-theory). Considering the equations of plane singularities as superpotentials in Landau-Ginzburg theory, the L-functions can be then (conjecturally) interpreted as partition functions. The functional equation for these L-functions matches the duality for superpolynomials (DAHA and others; one of the most important symmetries in their theory). This seems a fundamental connection between physics (S-duality) and number theory. These polynomials satisfy Riemann Hypothesis for sufficiently small q (another interesting development), which is hopefully related to "phase transitions" in LGSM, at least by analogy with the Lee-Yang theorem from spin chains. All constructions will be provided; the L-functions above, called motivic superpolynomials, will be calculated in some simple cases.

Corrado de Concini

Projective Wonderful Models for Toric Arrangements and their Cohomology

Joint with Giovanni Gai.
I plan to sketch an algorithmic procedure which allows to build projective wonderful models for the complement of a toric arrangement in a n-dimensional algebraic torus T in analogy with the case of subspaces in a linear or projective space. The main step of the construction is a combinatorial algorithm that produces a projective toric variety in which the closure of each layer of the arrangement is smooth.
The explicit procedure of our construction allows us to describe the integer cohomology rings of such models by generators and relations and introduce a "ring of conditions" for any toric arrangement.

Benjamin Enriquez

Double shuffle and associator relations for multiple zeta values

We explain how the double shuffle relations between MZVs can be formulated in terms of infinitesimal braids. This enables one to construct a bitorsor structure on the scheme defined by these relations and also to give a new proof of the inclusion in this scheme of the scheme of associators (joint w. H. Furusho).

Pavel Etingof

Short star-products for filtered quantizations

Let A be a filtered Poisson algebra with Poisson bracket $\{,\}$ of degree -2. A *star product* on A is an associative product $*: A \otimes A \to A$ given by

$$a * b = ab + \sum_{i \geq 1} C_i(a, b),$$

where C_i has degree $-2i$ and $C_1(a, b) - C_1(b, a) = \{a, b\}$. We call the product $*$ *even* if $C_i(a, b) = (-1)^i C_i(b, a)$ for all i, and call it *short* if $C_i(a, b) = 0$ whenever $i > \min(\deg(a), \deg(b))$.
Motivated by three-dimensional $N = 4$ superconformal field theory, In 2016 Beem, Peelaers and Rastelli considered short even star-products for homogeneous symplectic singularities (more precisely, hyperKähler cones) and conjectured that that they exist and depend on finitely many parameters. We prove the dependence on finitely many parameters in general and existence for a large class of examples, using the connection of this problem with zeroth Hochschild homology of quantizations suggested by Kontsevich.
Beem, Peelaers and Rastelli also computed the first few terms of short quantizations for Kleinian singularities of type A, which were later computed to all orders by Dedushenko, Pufu and Yacoby. We will discuss some generalizations of these results.
This is joint work with Eric Rains and Douglas Stryker.

Philippe Di Francesco

Triangular Ice Combinatorics

Alternating Sign Matrices (ASM) are at the confluent of many interesting combinatorial/algebraic problems: Laurent phenomenon for the octahedron equation, configurations of the Square Ice (Six Vertex model), Descending Plane Partitions (DPP), etc. Here we consider the Triangular Lattice version of the Ice model with suitable boundary conditions leading to an integrable 20 Vertex model. Configurations give rise to generalizations of ASM, which we coin Alternating Phase Matrices (APM). We generalize the ASM-DPP correspondence by showing that APM are equinumerous to the quarter-turn symmetric domino tilings of a quasi-Aztec square with a central cross-shaped hole, and obtain a compact determinant formula for their enumeration. We also present conjectures for triangular Ice with other types of boundary conditions, and preliminary results on the shape of large APM.
(joint work with E. Guitter, IPhT Saclay, France).

Jürg Fröhlich

Bose Gases and Loop Ensembles

I sketch some recent results on the equilibrium statistical mechanics of non-relativistic Bose gases, scalar euclidian field theories and self-avoiding walks - topics that were of interest to Giovanni Felder when he was my PhD student. I start by reviewing the "Ginibre-representation" of non-relativistic Bose gases in thermal equilibrium. I then sketch how the Symanzik-representation of some scalar euclidian field theories can be derived from the "Ginibre-representation" in the so-called mean-field limit, and how the theory of self-avoiding walks appears in the "de Gennes limit" where the number of species of bosons tends to 0. Time permitting, I sketch a form of cluster expansion enabling one to construct the thermodynamic limit of these systems. I present a novel (not entirely rigorous) argument explaining why the continuum limit of ϕ^4- theory in FOUR dimensions is trivial (Gaussian).

Toshitake Kohno

Higher holonomy and iterated integrals

The purpose of this talk is to explain a method to extend monodromy representations of flat connections to higher categories. We develop a method to construct representations of the infinity-category of homotopy infinity-groupoid of a manifold by means of K.-T. Chen's formal homology connections. In particular, we describe 2-holonomy maps for hyperplane arrangements and discuss higher category extensions of KZ connections. As an application we describe representations of the 2-category of braid cobordisms.

Antti Kupiainen

Constructive Liouville Theory

A. Polyakov introduced Liouville Conformal Field theory (LCFT) in 1981 as a way to put a natural measure on the set of Riemannian metrics over a fixed two dimensional manifold. Ever since, the work of Polyakov has echoed in various branches of physics and mathematics, ranging from string theory to probability theory through geometry. In the context of 2D quantum gravity models, Polyakov's approach is conjecturally equivalent to the scaling limit of Random Planar Maps and through the Alday-Gaiotto- Tachikava correspondence LCFT is conjecturally related to certain 4D Yang-Mills theories. Through the work of Dorn,Otto, Zamolodchikov and Zamolodchikov and Teschner LCFT is believed to be to a certain extent integrable. I will review a probabilistic approach to LCFT based on Kahane's theory of Gaussian Multiplicative Chaos developed together with David, Rhodes and Vargas. In particular this has recently led to proof of an integrability conjecture on LCFT, the celebrated DOZZ formula, in a joint work with Rhodes and Vargas.

Nikita Nekrasov

Omega/Mho backgrounds, Chern-Simons theory, and many-body systems

One of the interests of Giovanni was the connection between the many-body quantum mechanical systems and conformal field theory. I will review the old and new approaches to these questions, with the emphasis on the recent applications of supersymmetric gauge theories.

Andrei Okounkov

Characters and difference equations

Characters of Lie algebras and related algebras (both in zero and prime characteristic) fit into a larger class of special functions of, essentially, q-hypergeometric type, that is, solutions of certain regular q-difference equations. Basic phenomena of representation theory, like the appearance of a submodule under a specialization of parameters, have analytic counterparts in this broader setting. My goal in this talk is to explain the enumerative geometry perspective on both the representations and q-difference equations in question, following ideas from joint projects with Roman Bezrukavnikov and Mina Aganagic.

Catharina Stroppel

Some recent developments in super representation theory

In this talk I like to give some overview about aspects of the representation theory of super groups focusing on modern approaches and categorical aspects rather than the more traditional (sometimes rather ad hoc) Lie theoretic methods. Important inputs hereby are the Okounkov-Vershik approach (in a non-semsimple setting), Fock space combinatorics, quantum symmetric pairs and categorification methods. The goal of the talk will be a sketch how these techniques come together to get a better understanding of the representation theory of super groups. We will indicate how geometry comes into the picture and - if time allows - state a result which replaces the missing localisation theorem for super groups and allows to compute explicitly decomposition numbers.

Nicolai Reshetikhin

Spin Calogero-Moser type systems on moduli space of flat connections

n/a

Alexander Varchenko

Hyperelliptic integrals modulo p and Cartier-Manin matrices

The hypergeometric solutions of the KZ differential equations were constructed about 30 years ago. The polynomial solutions of the KZ equations over the finite field F_p with a prime number p elements were constructed recently. I will consider the example of the KZ equations whose hypergeometric solutions are given by hyperelliptic integrals of genus g. It is known that in this case the total $2g$-dimensional space of holomorphic solutions is given by the hyperelliptic integrals. It turns out that the recent construction of the polynomial solutions over the field F_p in this case gives a g-dimensional space of solutions, that is, a "half" of what the complex analytic construction gives. It turns out also that all the constructed polynomial solutions over the field F_p can be obtained by reduction modulo p of a single distinguished hypergeometric solution. The corresponding formulas involve the entries of the Cartier-Manin matrix of the hyperelliptic curve.

Michèle Vergne

Quiver Grassmannians, Q-intersection and Horn conditions

Work in Common with Welleda Baldoni, and Michael Walter.
Let Q be a quiver and $\mathcal{V} = (V_x)_{x \in \mathbb{Q}}$ be a collection of vector spaces. Let $v \in H_Q = \oplus_{a:x \to y} Hom(V_x, V_y)$ be a representation of Q and consider the space of subrepresentations $S_x \subset V_x$ of v with $dim(S_x) = \alpha(x)$. Let $\Omega = (\Omega_x)$ be a collection of Schubert varieties, with $\Omega_x \subset Gr(\alpha(x), V_x)$. We determine necessary and sufficient conditions on Ω in order that for any $v \in H_Q$, there exists a subrepresentation $S = (S_x)$ of v such that $S_x \in \Omega_x$. This result generalizes Schoffield conditions on general subdimension vectors as well as Horn conditions for intersection of Schubert varieties. We apply this result to obtain results on the decomposition of the space $Sym^*(H_Q)$ of polynomial functions on H_Q under the action of $\prod_x GL(V_x)$.

Chenchang Zhu

Higher groups in higher gauge theory

There has been much recent development on higher symmetries in topological orders as the study of topological phase of matters has become a very active field in condensed matter physics. In this talk, we will carry out the mathematical foundation of a recent joint project with Tian Lan and Xiao-Gang Wen in the above direction. Higher groups are group objects in a higher category. In a very concise way, they can be realised as a simplicial object satisfying suitable Kan conditions. We will give a more explicit algebraic model to realise some of them. This model is between the target of Dold-Kan functor and the skeleton case. These higher groups are then used in the sigma model as the target space to realise various physical meaningful phases.